An evolutionary approach for multi-objective vehicle routing problems with backhauls

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Abstract

The vehicle routing problem (VRP) is an important aspect of transportation logistics with many variants. This paper studies the VRP with backhauls (VRPB) in which the set of customers is partitioned into two subsets: linehaul customers requiring a quantity of product to be delivered, and backhaul customers with a quantity to be picked up. The basic VRPB involves finding a collection of routes with minimum cost, such that all linehaul and backhaul customers are serviced. A common variant is the VRP with selective backhauls (VRPSB), where the collection from backhaul customers is optional. For most real world applications, the number of vehicles, the total travel cost, and the uncollected backhauls are all important objectives to be minimized, so the VRPB needs to be tackled as a multi-objective problem. In this paper, a similarity-based selection evolutionary algorithm approach is proposed for finding improved multi-objective solutions for VRPB, VRPSB, and two further generalizations of them, with fully multi-objective performance evaluation.

Keywords: Vehicle routing problem, evolutionary computation, multi-objective optimization, combinatorial optimization.

1. Introduction

The main objective of the vehicle routing problem (VRP) is to obtain the lowest-cost set of routes to deliver demand to customers from a depot, and sometimes to also collect a quantity of product from customers. Since Dantzig and Ramser [1] introduced the VRP more than 50 years ago, it has been the subject

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of extensive research and has become one of the most studied combinatorial optimization problems. As observed by Golden et al. [2, Preface],

> "vehicle routing may be the single biggest success story in operations research. For example, each day 103,500 drivers at UPS follow computer-generated routes. The drivers visit 7.9 million customers and handle an average of 15.6 million packages",

so it is of tremendous practical importance for transportation logistics. In fact, because of the diversity of operating rules and constraints encountered in real-world applications, numerous variants of the problem exist, and the VRP should really be viewed as a whole class of problems [3].

One particularly common variant is the VRP *with backhauls* (VRPB), which involves both delivery and collection points [4]. *Linehaul* customers are sites with a demand of goods, and deliveries have to be made to them from the depot or distribution center. *Backhaul* customers are points from which a quantity of goods has to be collected and taken to the depot. A practical example of this is the manufacturing industry, where factories are the linehaul customers, and raw materials and components are supplied by the backhaul customers.

The general problem consists of designing a set of routes with minimum cost to service the given linehaul and backhaul customers. Since the VRP was originally proposed as a generalization of the traveling salesman problem [1], the cost has primarily been associated with the number of routes (or vehicles) and the total travel distance (or time), but there are several other potential sources of cost [5]. In practice, given the constraints, minimization of the travel cost often results in an increased number of routes, so if both objectives are considered to be of importance, the VRPB really needs to be tackled as a bi-objective problem. Moreover, if a revenue is associated with each backhaul customer, and these are considered optional, that is another objective which needs to be taken into account, and the VRPB should be tackled as a tri-objective problem. This last variant is known as VRP with *selective backhauls* (VRPSB) [6].

Exact methods have been devised to find optimal solutions for relatively small instances of the VRPB [7, 8], but, since it belongs to the NP-hard class of problems [9], the computation time required increases considerably for larger instances. For realistically sized problems, one is therefore forced to use heuristic approaches. There have been many past studies which have solved the VRPB as a single-objective problem using heuristic and metaheuristic methods, such as tabu search [10, 11] and ant colony algorithms [12]. However, very few studies have considered the VRPB and VRPSB as multi-objective problems. One particularly effective approach, that has not been fully investigated before, involves using evolutionary algorithms (EAs) which can generate a whole population of solutions to cover the full range of trade-offs among objectives.

In a preliminary study, a simple evolutionary algorithm for solving standard benchmark instances of the VRPB and VRPSB variants was introduced with promising results [13]. This paper now presents an improved evolutionary approach, involving the optimization of two and three objectives for well-known instances of the VRPB and two further generalizations of the problem. It builds on an earlier application of evolutionary computation techniques to the VRP *with time windows* [14], that introduced a novel selection process involving solution dissimilarity to generate solution sets with better coverage of the full range of trade-off possibilities. However, application to the VRPB is not straightforward, because it requires the formulation of supplementary problem-specific evolutionary operators, and a careful multi-objective evaluation of the solutions generated. Comparisons with existing single-objective algorithms are first provided, and then fully multi-objective performance metrics are used to explore the properties of the current benchmark instances, and demonstrate the advantages of the VRPB-specific similarity-based selection processes over the general purpose crowding mechanism of the widely used NSGA-II [15] and over the decomposition approach of the successful MOEA/D [16]. Moreover, the multi-objective performance is further analyzed by studying the performance of the algorithm when different objectives are considered for optimization.

The remainder of this paper is organized as follows: The next section describes formally the main VRPB variants, and Section 3 surveys the principal previous studies of them. Section 4 reviews the key concepts of multi-objective optimization, and describes the multi-objective performance metric used later. The proposed approach for solving the VRPB as a multi-objective problem, and its extension for solving VRPSB, are described in Section 5. Then, Section 6 presents results from the proposed algorithm for a range of benchmark problem instances, and provides comparisons with previously published algorithms. Finally, some conclusions are provided in Section 7.

2. VRP with Backhauls

The basic version of the VRP is the *capacitated* VRP (CVRP), which considers a set $\mathcal{V} = \{0, \ldots, N\}$ of $N + 1$ vertices, where the subset $\mathcal{V}' = \mathcal{V} \setminus \{0\} = \{1, \ldots, N\}$ are the customers. Each customer $i \in \mathcal{V}'$ is geographically located at coordinates (x_i, y_i) and has a demand of goods $d_i > 0$ to be delivered. The special vertex 0, located at (x_0, y_0), with $d_0 = 0$, is the *depot* from which the customers are serviced. There is a homogeneous fleet of K vehicles available to deliver demand to customers, departing from and arriving at the depot, and having capacity $Q \geq \max\{d_i : i = 1, \ldots, N\}$. The travel from vertex i to vertex j has an associated cost c_{ij}, and the core problem consists of finding a set of routes which minimizes the total travel cost.

The VRP *with backhauls* (VRPB) is an extension of the CVRP, where the customers are grouped into *linehaul* customers, which have a demand of goods, and *backhaul* customers, from which a quantity of goods has to be collected. Thus, an instance of the VRPB can be formally defined as a set $\mathcal{V} = \{0, \ldots, N_L, N_L + 1, \ldots, N_L + N_B\}$ of $N + 1$ vertices, representing the depot and $N = N_L + N_B$ customers [7]. The customers are represented by the vertices in subset $\mathcal{V}' = \mathcal{V} \setminus \{0\} = \{1, \ldots, N_L, N_L + 1, \ldots, N_L + N_B\}$, and each customer $i \in \mathcal{V}'$ is geographically located at coordinates (x_i, y_i). The subset $\mathcal{V}_L = \{1, \ldots, N_L\}$ corresponds to linehaul customers, where each customer $i \in \mathcal{V}_L$ has a demand of goods $d_i > 0$ to be delivered. The subset $\mathcal{V}_B = \{N_L + 1, \ldots, N_L + N_B\}$

represents the backhaul customers, where each customer $i \in \mathcal{V}_B$ has a supply $s_i > 0$ to be collected. A homogeneous fleet of K vehicles is available to deliver and collect goods to and from customers, departing from and arriving at the depot, and having capacity $Q \geq \max\{\max\{d_i : i \in \mathcal{V}_L\}, \max\{s_i : i \in \mathcal{V}_B\}\}$.

The main objective is to find a set of K routes which minimize the total travel cost, subject to the following conditions [17]:

i) each vehicle services exactly one route,

ii) each customer is visited exactly once by one vehicle,

iii) a route is not allowed to consist entirely of backhaul customers,

iv) backhaul customers in a route can only be served after all linehaul customers, and

v) for each route, the total load associated with linehaul or backhaul customers cannot exceed the vehicle capacity Q.

The fourth constraint corresponds to the fact that most vehicles are rear-loaded and rearrangement of vehicle loads at delivery points is generally deemed infeasible [18], and also accommodates the fact that linehaul customers frequently prefer early deliveries, while backhaul customers prefer late collections [19].

Some interesting practical generalizations of the VRPB involve relaxing the third and fourth constraints. One of them is known as the VRP *with mixed backhauls* (VRPMB), which allows backhaul customers to be serviced at any point within a route. That is, linehaul and backhaul customers can be mixed freely within a route, and routes can consist only of backhaul customers. Another variation is the VRP *with simultaneous deliveries and pickups* (VRPSDP), in which customers simultaneously demand goods from and supply goods to the depot. In this case, both delivery and pickup should occur at customers, and they should be performed simultaneously so that each customer is only visited once by a vehicle, and unloading is obviously done before loading.

There are further generalizations of these problems, where all the linehaul customers must be visited, but picking up from backhaul customers is optional. These are the VRP *with selective backhauls* (VRPSB) [6], and the VRP *with mixed and selective backhauls* (VRPMSB). In these problems, each backhaul customer $i \in \mathcal{V}_B$ has an associated profit $p_i > 0$, and consequently

$$P = \sum_{i \in \mathcal{V}_B} p_i \tag{1}$$

is the total possible profit. The VRPSB and VRPMSB can simply involve determining a set of vehicle routes with minimum net cost (i.e., routing cost minus collected profit), given that visiting backhaul customers is optional. However, since customer satisfaction is important for service providers, it is often useful to optimize the routing cost and profit collection objectives separately, and consider the trade-off. The remaining SB combination, VRPSDP with *selective backhauls*, is easier, since all backhaul customers are automatically visited because they are also linehaul customers.

If, for any of the VRPB variants described above, the cardinality of the set of routes is not pre-specified to equal the number of available vehicles K, minimizing the number of routes (and hence number of drivers) can be treated as yet another optimizable objective. Thus, if all the objectives are considered important, these problems will need to be considered as multi-objective, and there will be three objective functions to minimize simultaneously: (*i*) the number of routes/vehicles, (*ii*) the travel cost, and (*iii*) the uncollected revenue.

For each of VRPB, VRPMB, VRPSDP, VRPSB and VRPMSB, the aim is to find lowest-cost sets of routes $\mathcal{R} = \{r_1, \ldots, r_k\}$, such that each route begins and ends at the depot, and the further constraints of the given problem are satisfied. To proceed, a formal specification of the costs is needed.

Let $r_j = \langle u(1,j), \ldots, u(N_j,j)\rangle$ specify the sequence of N_j customers serviced in the j-th route, where $u(i,j)$ is the i-th customer to be visited in the j-th route, and $u(0,j) = u(N_j+1,j) = 0$ is the depot. Then $\mathcal{V}_j = \{u(1,j), \ldots, u(N_j,j)\}$ is the set of customers in the j-th route. The travel cost c_{ab} between customers a and b might be the distance (or associated fuel cost), or travel time (or associated driver cost), or some combination. The following analysis is not affected by exactly how the travel cost is defined for particular problem instances. The total travel cost C_j for the j-th route is simply the sum

$$C_j = \sum_{i\,=\,0}^{N_j} c_{u(i,j)u(i+1,j)} \ , \tag{2}$$

and the profit P_j from collected loads in the j-th route will be

$$P_j = \sum_{i\,=\,1}^{N_j} p_{u(i,j)} \ , \tag{3}$$

where $p_{u(i,j)} = 0$ if $u(i,j) \in \mathcal{V}_L$.

The three objective functions that this study will aim to optimize are the number of routes or vehicles used

$$f_1(\mathcal{R}) = |\mathcal{R}| = k \ , \tag{4}$$

the total travel cost

$$f_2(\mathcal{R}) = \sum_{j=1}^{k} C_j \ , \tag{5}$$

and the total uncollected profit

$$f_3(\mathcal{R}) = P - \sum_{j=1}^{k} P_j \ , \tag{6}$$

subject to the constraints explained earlier.

3. Previous studies

A number of existing approaches for solving VRPB, VRPMB and VRPSDP variants have already been published, and Parragh [20] provides an excellent survey of them. The key studies and recent advances will be reviewed here.

Most previous approaches have involved heuristic methods. For example, Toth and Vigo [21] presented a cluster-first-route-second heuristic for solving the VRPB, which uses a clustering method and may also be used for solving problems with an asymmetric cost. Their approach exploited the information of the normally infeasible VRPB solutions associated with a lower bound, using a Lagrangian relaxation based bound that they had developed previously. The final set of feasible routes was built using a modified Traveling Salesman Problem heuristic, with inter-route and intra-route arc exchanges.

More recently, Ropke and Pisinger [19] surveyed models of the backhaul constraints, and introduced a unified model that is capable of handling the VRPB, VRPMB and VRPSDP. Their model can be seen as a general problem, which can be solved through an improved version of the large neighborhood search heuristic previously proposed by them. Their results were comparable to, or improved upon, those found by earlier heuristics for these variants of the problem, and they are still responsible for almost all best-known solutions for the VRPMB benchmark instances.

Metaheuristic approaches have also been used for solving VRPB problems. For example, Osman and Wassan [22] proposed two route-construction heuristics to generate initial solutions quickly, which were then improved by a reactive tabu search metaheuristic. The reactive concept was used in a new way to select different neighborhood structures for the intensification and diversification phases of the search. Brandão [23] presented a tabu search algorithm that, starting from pseudo-lower bounds, was able to match almost all the best published solutions and also found many new best solutions, for a large set of benchmark problems. Gajpal and Abad [24] proposed a multi-ant colony system for solving the VRPB that used a new construction rule as well as two multi-route local search schemes.

For the VRPMB, Salhi and Nagy [25] proposed an extension of the load based insertion procedure of Golden et al. [26] that considered clusters of backhaul customers, instead of single backhaul customers. For the first time, the multi-depot case was also tackled. This extension was accommodated by the notion of borderline customers, i.e. customers situated approximately half-way between two depots. The procedure divided the set of linehaul customers into borderline and non-borderline. First, the non-borderline customers are assigned to their nearest depot and the corresponding VRP is solved, and then the borderline customers are inserted one-by-one into the routes.

Nagy and Salhi [27] elaborated on an integrated construction-improvement heuristic for both the VRPMB and VRPSDP. Their procedure starts from a weakly feasible VRPMB solution, i.e. one that does not violate the maximum route length, nor have the total load picked up or delivered exceed the vehicle's capacity. Strong feasibility is then attained when the load constraint is respected at every arc. First, the weakly feasible solution is improved by local search procedures, and then the improved solution is made strongly feasible and improved further while retaining strong feasibility.

Crispim and Brandão [28] also tackled VRPMB and VRPSDP, and presented a hybrid algorithm involving both tabu search and the variable neighborhood search metaheuristics. The search here is first carried out in one of the neighborhood structures using moves which are not tabu during a certain time, and then the neighborhood structure is changed consecutively until the stop criterion is reached. This combined use of two metaheuristics was an attempt to obtain more diversification in the search, and hence wider coverage of the solution space. Dethloff [29] proposed an extension of the cheapest insertion heuristic for solving the VRPSDP that not only relies on the measure of travel cost, but also on residual capacity and radial surcharge. That method was also used to solve the VRPMB. Montané and Galvão [30] discussed another tabu search algorithm for solving the VRPSDP. They combined the four construction methods used by Gendreau et al. [31] with a tour partitioning heuristic and an adapted sweep algorithm to generate an initial solution, resulting in eight different methods. Four different neighborhoods were implemented: a relocation, an interchange, a crossover, and a combined neighborhood. At every iteration, the best feasible non-tabu solution of the neighborhood was chosen, and the 2-opt operator used to improve the solution found. Bianchessi and Righini [32] compared a tabu search algorithm to different construction and improvement heuristics. A combination of various arc-exchange (involving two or three routes) and node-exchange (relocate, exchange) neighborhoods were tested. The tabu search algorithm used two tabu lists, one for arc-based and one for node-based neighborhoods.

More recently, Assis et al. [33] proposed a multi-objective iterated local search approach for solving the VRPSDP, with simultaneous minimization of routing cost and uncollected profit, and used multi-objective performance metrics to compare their results with other approaches. Jun and Kim [34] introduced a heuristic algorithm for both VRPMB and VRPSDP that consisted of a route construction procedure, a route improvement procedure, and a solution perturbation procedure. A sweep-based route construction method was developed to generate initial solutions. Then a series of inter- and intra-route improvement algorithms were applied in the route improvement procedure. The perturbation procedure operates, by removing and reinserting routes and customers from solutions, to escape from local optima. Goskal et al. [35] presented a heuristic approach based on particle swarm optimization, in which a local search is performed by a variable neighborhood descent algorithm. It implements an annealing-like strategy to preserve the swarm diversity. Finally, Subramanian et al. [36] proposed a branch-cut-and-price approach for the VRPMB and VRPSDP. The algorithm was tested on well-known benchmark instances and some lower bounds were improved.

With the exception of the recent study of Assis et al. [33], all the approaches reviewed above have solved the VRPB, VRPMB and VRPSDP as single-objective problems that only minimized the total travel cost, i.e. $f_2(\mathcal{R})$ in equation (5). In fact, there appear to be no other previous studies treating the solution of these problems as multi-objective. Furthermore, the *selective backhauls* versions appear to have never before been the subject of systematic study. This led Garcia, in a preliminary report [13], to rectify these omissions by proposing a multi-objective evolutionary algorithm for solving the VRPB and

VRPSB, involving the minimization of the number of routes, the routing cost and the uncollected profit. This paper will extend and complete the preliminary work of Garcia [13].

4. Multi-objective optimization

To proceed, a more formal problem specification is required. Using standard notation and terminology [37], any multi-objective optimization problem can, without loss of generality, be defined as the minimization problem

$$\text{minimize } \mathbf{f}(\mathbf{x}) = (f_1(\mathbf{x}), \ldots, f_F(\mathbf{x})) \tag{7}$$

subject to the constraints

$$g_i(\mathbf{x}) \leq 0 \qquad \forall\, i = 1, \ldots, p\ , \tag{8}$$

$$h_j(\mathbf{x}) = 0 \qquad \forall\, j = 1, \ldots, q\ , \tag{9}$$

where $\mathbf{x} \in \mathcal{X}$ is a solution to the problem, $\mathcal{X}$ is the solution space, and $f_i : \mathcal{X} \to \mathbb{R}$, for $i = 1, \ldots, F$, are F objective functions. The constraint functions $g_i, h_j : \mathcal{X} \to \mathbb{R}$ in (8) and (9) restrict $\mathbf{x}$ to a feasible region $\mathcal{X}' \subseteq \mathcal{X}$.

A solution $\mathbf{x} \in \mathcal{X}$ is said to *cover* the solution $\mathbf{y} \in \mathcal{X}$, written as $\mathbf{x} \preceq \mathbf{y}$, if $f_i(\mathbf{x}) \leq f_i(\mathbf{y})$, $\forall\, i \in \{1, \ldots, F\}$. Solution $\mathbf{x}$ *dominates* solution $\mathbf{y}$, written as $\mathbf{x} \prec \mathbf{y}$, if and only if $\mathbf{x} \preceq \mathbf{y}$ and $\exists\, j \in \{1, \ldots, F\} : f_j(\mathbf{x}) < f_j(\mathbf{y})$. Consequently, a solution $\mathbf{x} \in \mathcal{S} \subseteq \mathcal{X}$ is *non-dominated* with respect to $\mathcal{S}$ if there is no solution $\mathbf{y} \in \mathcal{S}$ such that $\mathbf{y} \prec \mathbf{x}$. A solution $\mathbf{x} \in \mathcal{X}$ is said to be *Pareto optimal* if it is non-dominated with respect to $\mathcal{X}$, and the *Pareto optimal set* is defined as $\mathcal{P}_s = \{\mathbf{x} \in \mathcal{X} \mid \mathbf{x} \text{ is Pareto optimal}\}$. Finally, the *Pareto front* is defined as $\mathcal{P}_f = \{\mathbf{f}(\mathbf{x}) \in \mathbb{R}^F \mid \mathbf{x} \in \mathcal{P}_s\}$. The aim of the optimization process is to find the best representation of the Pareto front for the given problem instance.

4.1. Multi-objective performance metrics

In contrast to single-objective problems, where one can straightforwardly compare the best solutions from the various approaches studied, multi-objective problems require comparison of whole sets of solutions. This is because the task of approximating the Pareto optimal set not only involves minimizing the distance of the generated non-dominated solutions to the Pareto optimal set, but also maximizing the diversity of the achieved Pareto set approximation, so that similar solutions in the resulting set are avoided and a good representation of the trade-offs between objectives is obtained [38]. For this reason, the definition and use of appropriate multi-objective performance metrics or indicators is crucial.

Numerous performance indicators have already been proposed in the literature [39, 40, 41]. However, many of them are not *Pareto compliant*, which means that it cannot be concluded that one optimizer is better than another based only on them. One of the indicators that *is* Pareto compliant is the *hypervolume* metric $\mathcal{H}(\mathcal{A}, \mathbf{z})$ [42], which measures the size of the objective space defined by the approximation set $\mathcal{A}$ of solutions and a suitable reference point

$\mathbf{z}$. The idea is that a greater hypervolume indicates that the approximation set offers a closer approximation to the true Pareto front.

Formally, for a two-dimensional objective space $\mathbf{f}(\mathbf{x}) = \big(f_1(\mathbf{x}), f_2(\mathbf{x})\big)$, each solution $\mathbf{x}_i \in \mathcal{A}$ delimits a rectangle defined by its coordinates $\big(f_1(\mathbf{x}_i), f_2(\mathbf{x}_i)\big)$ and the reference point $\mathbf{z} = (z_1, z_2)$, and the size of the union of all such rectangles delimited by the solutions is used as the measure. This idea can be extended to any number of dimensions F to give the general hypervolume metric:

$$\mathcal{H}(\mathcal{A}, \mathbf{z}) = \lambda\left(\bigcup_{\mathbf{x}_i \,\in\, \mathcal{A}} \big\{[f_1(\mathbf{x}_i), z_1] \times \cdots \times [f_F(\mathbf{x}_i), z_F]\big\}\right) \,, \tag{10}$$

where $\lambda(\cdot)$ is the standard Lebesgue measure [43].

For maximization problems, it is common to take $\mathbf{z}$ to be the origin, while for minimization problems, $\mathbf{z}$ is set to equal or exceed the maximal values of each objective. Either way, when using this metric to compare the performance of two or more algorithms, the one providing solutions with the largest delimited hypervolume is regarded to be the best.

5. Multi-objective evolutionary algorithm for solving VRPBs

This paper deals with several different variants of the VRPB, so the proposed similarity-based selection multi-objective evolutionary algorithm (SSMOEA) approach is explained in two stages: first the algorithm for solving the basic VRPB is described, and then the various adaptations required for each of the specific variations.

5.1. SSMOEA for solving VRPB

As with all EA approaches, the basic idea is to maintain a population of individual problem solutions, and evolve them by natural selection of the fittest. The proposed SSMOEA builds on an earlier approach for solving the multi-objective VRP *with time windows* (VRPTW) [14]. The following defines the whole approach, with particular reference to the differences from the VRPTW study that relate to the absence of time windows and introduction of backhauls.

5.1.1. Solution representation

The VRPB solutions are simply lists of routes, which are themselves lists of customers, so each solution can be represented directly as a list of lists. A given route is encoded as a list of customer identifiers in the order they are serviced, and the solutions are encoded as lists of routes.

5.1.2. Random initial population

As usual with EAs, the process of solution evolution starts with an initial population [44] of randomly generated feasible solutions.

First, a random permutation of the backhaul customer identifiers is generated. Then, K routes are created serving exactly one backhaul customer each, and the remaining backhaul customers are assigned sequentially to the routes. When the next backhaul customer cannot be assigned to a given route due to the capacity constraint, it and the remaining backhaul customers are assigned to the next route, and so on. If, after assigning backhaul customers to the K-th route, there are still unassigned backhaul customers, further routes are created.

Then, a random permutation of the linehaul customer identifiers is generated. One linehaul customer is allocated to each of the previously generated routes, and the remaining linehaul customers are assigned to the existing routes using the same procedure as for the backhaul customers.

5.1.3. Solution fitness measure

As always with EAs, during the evolution, a measure of fitness for each individual solution is required to drive the natural selection process. For single-objective problems, the single objective function can be used to assign a fitness to each individual. For multi-objective problems, however, there is no single objective function that can be used as the fitness, and a whole set of solutions is required to represent the trade-offs among the objectives.

The individuals are fitness ranked in the multi-objective case here by using the non-dominance sorting criterion [45], whereby the population is divided into non-dominated fronts, and the depth of each front determines the fitness of the individuals in it. This follows the specific algorithm of Deb et al. [15], which is used in their NSGA-II algorithm.

5.1.4. Solution similarity measure

For multi-objective problems, it is important that the solutions in the final evolutionary population represent the full Pareto front, and not just a small portion of it. In fact, population diversity is important for EAs more generally, to avoid premature convergence and balance of the trade-off between exploration and exploitation of the search space [44].

A method is therefore required for evaluating solution *spread*, and using it to boost diversity generally requires the development of representation-specific tools. To accomplish this for the VRPB representation, a similarity measure was designed based on Jaccard's similarity coefficient, which computes the similarity of two sets as the ratio of the cardinalities of the intersection and the union of those sets [46]. Formally, the similarity of two sets A and B is

$$J(A,B) = \frac{|A \cap B|}{|A \cup B|} \ , \tag{11}$$

which is 1 if both sets contain the same elements, and 0 if their intersection is empty.

The easiest way to implement this measure for the VRPB is to consider each solution $\mathcal{R}$ as the union of its set of segments or arcs $(u(i,j), u(i+1,j))$, so

$$\mathcal{R} = \bigcup_{j=1}^{k} \bigcup_{i=0}^{N_j} \{(u(i,j), u(i+1,j))\} \ , \tag{12}$$

and the similarity of two solutions is the ratio of the number of arcs common to both solutions and the total number of arcs used by them. Thus, if $y_{ab\mathcal{R}} = 1$ when arc (a, b) is traversed by any vehicle in solution $\mathcal{R}$, and 0 otherwise, the similarity $s_{\mathcal{RQ}}$ between solutions $\mathcal{R}$ and $\mathcal{Q}$ is

$$s_{\mathcal{RQ}} = \sum_{a,b \,\in\, \mathcal{V}} y_{ab\mathcal{R}} \cdot y_{ab\mathcal{Q}} \Bigg/ \sum_{a,b \,\in\, \mathcal{V}} \operatorname{sign}\left(y_{ab\mathcal{R}} + y_{ab\mathcal{Q}}\right) . \tag{13}$$

In this, arcs (a, b) and (b, a) are considered to be different, even if their cost is the same.

As suggested by its name, the proposed SSMOEA also requires a measure of how similar a given solution is to the rest of the population. If $\boldsymbol{\mathcal{P}}$ is the population of solutions, and $|\boldsymbol{\mathcal{P}}| = M$ is the population size, the similarity $\mathcal{S}_{\mathcal{R}}$ of solution $\mathcal{R} \in \boldsymbol{\mathcal{P}}$ with the rest of the solutions in $\boldsymbol{\mathcal{P}}$ can be computed as simply the average similarity of $\mathcal{R}$ with every other solution $\mathcal{Q} \in \boldsymbol{\mathcal{P}}$, that is

$$\mathcal{S}_{\mathcal{R}} = \frac{1}{M-1} \sum_{\mathcal{Q} \,\in\, \boldsymbol{\mathcal{P}} \backslash \{\mathcal{R}\}} s_{\mathcal{RQ}} . \tag{14}$$

Finally,

$$\mathcal{D}(\boldsymbol{\mathcal{P}}) = 1 - \frac{1}{M} \sum_{\mathcal{R} \,\in\, \boldsymbol{\mathcal{P}}} \mathcal{S}_{\mathcal{R}} , \tag{15}$$

provides a measure of the diversity of the population of solutions $\boldsymbol{\mathcal{P}}$ that will be used later to evaluate how well the algorithms really are maintaining diversity.

5.1.5. Selection of parents

During any evolutionary process, individual solutions need to be selected from the population to be the *parents* that undergo recombination to create *offspring* to populate the next generation. A stochastic function is required such that the fittest individuals are most likely to be selected, but low-fitness individuals should also be given a small chance, so the algorithm is not too greedy. A standard binary tournament method [47] is used that randomly chooses two individuals from the population and selects the best to be a parent.

The SSMOEA proposed here has a crucial difference to most other EAs in that parent selection is not only based on their fitness, but also on the above similarity measure. The first of the two parents of each offspring is chosen on the basis of highest fitness, as usual, but the second is chosen on the basis of lowest similarity, with the aim of maintaining a high population diversity.

5.1.6. Recombination process

Recombination is the EA process of generating offspring from the selected parents, preferably in a way that combines and maintains the desirable features from both parents. The VRPB recombination operator here is designed to randomly select and preserve routes from both parents. First, a random number of routes are copied from the first parent into the offspring. Then all the routes from the second parent which are not in conflict with customers already copied from the first, are copied into the offspring. Finally, any unassigned customers are allocated, in the order they appear in the second parent, to the route where the lowest travel cost is achieved. If there are any remaining customers that cannot be inserted into the existing routes without violating a constraint, a new route is created.

5.1.7. Mutation operators

As is common in EAs, each new offspring also has a further stochastic change or *mutation*. For the VRPB, the proposed SSMOEA generates these with three operators that use three basic functions. The functions are:

`selectRoute` Stochastically selects a route according to the ratio of the travel cost to the number of customers, such that routes with a larger travel cost and fewer customers are more likely to be selected.

`selectCustomer` Stochastically selects one customer from a specific route according to the average length of its inbound and outbound arcs, such that customers with longer associated travel costs are more likely to be chosen. For the first and last customers in a route, only the outbound and inbound arcs, respectively, are taken into account.

`insertCustomers` Attempts to insert, one at a time, a set of customers into a specific route, where the lowest travel cost is obtained. When no route is specified, all existing routes are tested.

and these are used by the mutation operators as follows:

Reposition Moves a customer within a route. First `selectCustomer` chooses one customer from the route, and then `insertCustomers` reinserts it into the same route somewhere else.

Reallocation Takes a number of customers from a given route and allocates them to another. First, `selectCustomer` chooses two customers from the route, which are removed from the route along with all the customers in between them. Then, `insertCustomers` attempts to reallocate them into any of the existing routes, including the one they were removed from.

Exchange Swaps sequences of customers between two routes chosen with function `selectRoute`. First, `selectCustomer` chooses two customers from each route, which are removed from their route along with all the customers in between them. Then `insertCustomers` attempts to reallocate them into the other route. If one or more customers cannot be inserted into the other route, the original routes are preserved.

All offspring are subject to mutation. First, `selectRoute` chooses two routes. If they are the same, *Reallocation* is applied, otherwise, *Exchange* is applied. Then `selectRoute` chooses another route, and *Reposition* is applied.

5.1.8. Solution repair process

It is possible that offspring resulting from the recombination and mutation processes do not satisfy the relevant VPRB constraints. In particular, they may include routes serving backhaul customers only. If such non-feasible solutions exist, they must be submitted to a repair process. The proposed SSMOEA begins this process by identifying the non-feasible routes. Then, one linehaul customer is randomly chosen and removed from a feasible route, taking care that the route remains feasible, and inserted into one of the non-feasible routes. This process is repeated for every non-feasible route.

5.1.9. Selection for survival

For each new generation throughout evolution, individuals must be selected from the parents and offspring to survive [44]. The obvious choices are: the offspring population, a random selection from the combined parent and offspring populations, or the best individuals from the combined population. The first two approaches allow too many good-quality individuals to be lost, so the third approach is adopted for the proposed SSMOEA. The offspring and parent populations are combined and the solutions with the highest fitness, i.e. falling in the outermost non-dominated fronts, are taken to survive and form the next generation. If the population size is exceeded by taking the whole of the last selected front, the least similar solutions on that front are chosen.

5.1.10. Iteration and termination

The evolutionary processes of parent selection, offspring generation and survival selection are repeated for a fixed maximum number of generations, or until the the evolution has stabilized, as indicated by the diversity of solutions in the Pareto approximation not changing for a certain number of generations.

5.2. Adaptation for solving VRPMB and VRPSDP

For both the VRPMB and VRPSDP, the limitation of serving all the linehaul customers in a route first, before the backhaul customers, is relaxed so that any customer can be visited at any point within a route. Also, for the VRPMB, routes are allowed to consist of only backhaul customers. The SSMOEA was designed so that the only stage in the basic version above that needs adapting to accommodate these variations is the generation of the initial population.

5.2.1. Revised random initial population

The initial solutions here can be created by the following simplified process: A random permutation of all customers, linehaul and backhaul, is generated. The first customer in the permutation is taken to be the first location visited on the first route. Then, if the capacity is not exceeded, the next customer is placed on the current route after the previous customer. Otherwise, a new route is created with this customer as the first location on it to be visited. This process is repeated until all customers have been assigned to a route.

5.3. Adaptation for solving VRPSB and VRPMSB

For the selective backhaul variants, VRPSB and VRPMSB, the uncollected profit, i.e. $f_3(\mathcal{R})$ in equation (6), is minimized too. This means solutions are needed with different collected profits, and hence variety in the number of backhaul customers. To accomplish this, the SSMOEA procedures for generating the initial solutions and mutations need modification.

5.3.1. Revised random initial population

The original initial population generation process above only needs a slight modification. Instead of starting with a random permutation of all N_B backhaul customers, a random integer $n_B \in [0, N_B]$ is chosen, and that number of backhaul customers is randomly selected and permuted.

5.3.2. Revised mutation process

In addition to the three mutation operators of the basic VRPB, one further operator is required:

Modify_n_B Randomly chooses whether to insert or remove a backhaul customer, except when $n_B = 0$ or $n_B = N_B$, in which case a backhaul customer is added or removed, respectively. If a backhaul customer is to be removed, one route is randomly selected with `selectRoute` and one backhaul customer on it is chosen at random and removed. If a backhaul customer is to be inserted, one of the backhaul customers which are not present in the solution is randomly chosen, and `insertCustomers` is used to insert that customer into the route where the lowest cost is obtained.

The overall mutation process is then modified to become: First, `selectRoute` chooses two routes. If they are the same, *Reallocation* is applied and followed by *Modify_n_B*, otherwise, *Exchange* is applied. Then `selectRoute` chooses another route, and *Reposition* is applied.

6. Experimental study

This study has three main purposes. First, to determine whether the proposed SSMOEA is still able to find best-known solutions for popular benchmark problem instances, even though it is set up to find whole Pareto approximations. To this end, SSMOEA was set to minimize simultaneously the number of routes and the total travel cost for VRPB, VRPMB and VRPSDP, with the results compared to existing approaches designed to optimize only the travel cost. Second, which is one of the main contributions of this study, to carry out a fully multi-objective performance evaluation of SSMOEA, for all problem variants, i.e. VRPB, VRPMB, VRPSDP, VRPSB and VRPMSB. That will include a comparison with the popular crowding mechanism of NSGA-II [15] and with the successful decomposition method of MOEA/D [16], to determine the advantage of introducing the similarity-based selection procedures. Third, to evaluate the ability of the proposed SSMOEA to find the same non-dominated solutions when it is set to optimize different objectives.

To enable reliable comparisons, three publicly available benchmark problem sets were used: that of Goetschalckx and Jacobs-Blecha [18], providing 62 VRPB instances of sizes $N = 25$ to 150, and the two sets of Salhi and Nagy [25], providing 36 VRPMB and 24 VRPSDP instances, with $N = 50$ to 150.

Given the complexity of the problem and the extremely large search space, the free parameters of SSMOEA were set to values that proved to work well in preliminary testing: The population size was set to equal the instance size. The number of generations of evolution was set to 50 times the product of the population size and the number of objectives to optimize, unless there was no change in the non-dominated solutions' diversity for 5% of the maximum number of generations. The SSMOEA, and the corresponding NSGA-II crowding version and the MOEA/D decomposition approach, were each run 30 times for each problem instance to provide reliable statistics.

6.1. Single-objective performance comparison with previous approaches

The initial aim was to compare the performance of the proposed SSMOEA with existing single-objective approaches. To accomplish this, SSMOEA was set to minimize the number of routes, objective $f_1(\mathcal{R})$ in equation (4), and the total travel cost, objective $f_2(\mathcal{R})$ in equation (5), simultaneously. That is, VRPB, VRPMB, and VRPSDP were considered bi-objective problems, but for each benchmark instance, only the SSMOEA solution with the required number of routes K and the lowest cost was used. Then, the average best costs from 30 repetitions were determined, and the percentage gaps between the best results from SSMOEA and the best-known results were calculated.

6.1.1. VRPB

Table 1 presents the details of the VRPB results. The first five columns show the properties of the benchmark instances, and the next two columns show the best-known result for each instance and the study that first obtained it. The following five columns show the results with K routes from SSMOEA: the first two show the best result and the percentage gap between this and the best-known result, and the next three show the average of the best results from each of the 30 repetitions, its corresponding standard deviation, and the percentage gap between the average best result and the best-known. The last two columns show the best results found by SSMOEA with $k < K$ routes.

SSMOEA was able to find the best-known solution for 25 out of the 62 instances, and the results for 28 of the remaining 37 instances are no more than 1.00% above the best-known. In fact, the average percentage gap between the best results from SSMOEA and the best-known results is only 0.41%. The average gap between the average best result found by SSMOEA and the best-known result is 2.67%. Interestingly, SSMOEA was also able to find solutions with a smaller number of routes k than the established K routes for 17 instances, and 11 of these solutions (shown in bold) also have a lower cost than the best-known,

Table 1: Best-known results compared to the results from SSMOEA on the Goetschalckx and Jacbos-Blecha [18] VRPB instances.

Instance					Best-known		Lowest cost with K routes					Lowest cost	
Id.	N	N_L	N_B	K	Cost	Ref	Cost	%	Avg.	Std.	%	k	Cost
A1	25	20	5	8	229884.00	[21]	229885.64	0.00	230422.42	733.99	0.23		
A2	25	20	5	5	180117.00	[21]	180119.21	0.00	181164.47	1118.00	0.58		
A3	25	20	5	4	163403.00	[21]	163405.36	0.00	165549.60	1996.65	1.31	**3**	**155796.39**
A4	25	20	5	3	155795.00	[21]	155796.39	0.00	158639.59	5308.95	1.83		
B1	30	20	10	7	239077.00	[21]	239080.18	0.00	239243.87	622.96	0.07		
B2	30	20	10	5	198045.00	[21]	198047.77	0.00	199600.55	2418.91	0.79		
B3	30	20	10	3	169368.00	[21]	169372.30	0.00	169845.07	1491.33	0.28		
C1	40	20	20	7	249448.00	[7]	250556.79	0.44	254587.26	2859.69	2.06		
C2	40	20	20	5	215019.00	[21]	215020.24	0.00	220591.68	3581.98	2.59		
C3	40	20	20	5	199344.00	[21]	199345.96	0.00	204674.21	5737.68	2.67	**4**	**195366.62**
C4	40	20	20	4	195365.00	[21]	195366.62	0.00	203846.89	5810.58	4.34		
D1	38	30	8	12	322530.00	[7]	322530.11	0.00	323714.91	1110.04	0.37	**11**	**317491.63**
D2	38	30	8	11	316708.86	[22]	317288.58	0.18	319602.50	2598.89	0.91		
D3	38	30	8	7	239478.63	[22]	239478.59	0.00	241091.31	2382.84	0.67		
D4	38	30	8	5	205831.94	[22]	205881.53	0.02	208833.94	1811.70	1.46		
E1	45	30	15	7	238879.58	[22]	238879.60	0.00	241544.10	2635.13	1.12		
E2	45	30	15	4	212262.00	[21]	212263.12	0.00	213662.03	2142.18	0.66		
E3	45	30	15	4	206658.00	[21]	206659.18	0.00	211052.77	4249.76	2.13		
F1	60	30	30	6	263173.00	[7]	268193.23	1.91	271062.65	2530.42	3.00		
F2	60	30	30	7	265213.00	[7]	265214.19	0.00	273930.41	4122.29	3.29		
F3	60	30	30	5	241120.00	[7]	241969.78	0.35	250572.37	5754.30	3.92		
F4	60	30	30	4	233861.00	[7]	235175.21	0.56	241078.77	4154.94	3.09		
G1	57	45	12	10	306305.00	[8]	306305.42	0.00	313806.69	5332.18	2.45		
G2	57	45	12	6	245440.99	[22]	245441.03	0.00	249214.99	6414.08	1.54		
G3	57	45	12	5	229507.00	[7]	229697.55	0.08	233846.50	3970.12	1.89		
G4	57	45	12	6	232521.00	[8]	232521.29	0.00	238241.10	4032.76	2.46	**5**	**229507.52**
G5	57	45	12	5	221730.00	[7]	221730.39	0.00	227333.34	6077.97	2.53	**4**	**218716.62**
G6	57	45	12	4	213457.00	[7]	213457.49	0.00	219365.55	3504.00	2.77		
H1	68	45	23	6	268933.00	[7]	268980.80	0.02	272804.92	2505.07	1.44		
H2	68	45	23	5	253365.00	[7]	253365.53	0.00	261565.69	5028.50	3.24		
H3	68	45	23	4	247449.00	[7]	247449.07	0.00	256087.49	5875.90	3.49		
H4	68	45	23	5	250220.77	[22]	250220.80	0.00	258795.38	4443.74	3.43	**4**	**247449.07**
H5	68	45	23	4	246121.00	[7]	246604.32	0.20	257070.31	5816.62	4.45		
H6	68	45	23	5	249135.00	[7]	249279.91	0.06	259414.63	5436.02	4.13	**4**	**246121.34**
I1	90	45	45	10	350245.28	[19]	350245.24	0.00	358762.69	5277.90	2.43	**9**	**347832.69**
I2	90	45	45	7	309943.00	[8]	311368.48	0.46	318375.42	4240.89	2.72		
I3	90	45	45	5	294507.00	[23]	294833.95	0.11	302436.56	6968.95	2.69		
I4	90	45	45	6	295988.00	[21]	297473.92	0.50	306645.51	5356.49	3.60	**5**	**294833.95**
I5	90	45	45	7	301226.00	[8]	306418.60	1.72	314014.49	4755.12	4.25	**5**	**294868.25**
J1	94	75	19	10	335006.00	[8]	336265.06	0.38	343608.26	3608.21	2.57		
J2	94	75	19	8	310417.00	[23]	310793.38	0.12	315184.57	2869.49	1.54		
J3	94	75	19	6	279219.00	[23]	282773.06	1.27	290717.52	4637.06	4.12		
J4	94	75	19	7	296533.00	[23]	297946.48	0.48	302132.35	3088.05	1.89		
K1	113	75	38	10	394071.16	[24]	394477.83	0.10	401983.32	3588.90	2.01		
K2	113	75	38	8	362130.00	[23]	363138.26	0.28	372859.89	5067.51	2.96		
K3	113	75	38	9	365693.00	[8]	367659.48	0.54	378115.72	5772.03	3.40	**8**	**364086.69**
K4	113	75	38	7	348949.39	[19]	353570.90	1.32	359881.06	4504.36	3.13		
L1	150	75	75	10	417896.72	[24]	426281.88	2.01	438645.11	6544.68	4.96		
L2	150	75	75	8	401228.00	[23]	401964.98	0.18	412245.15	8122.79	2.75		
L3	150	75	75	9	402677.72	[19]	404890.29	0.55	415921.23	6545.08	3.29	**8**	403678.84
L4	150	75	75	7	384636.33	[19]	386586.77	0.51	401274.79	7748.27	4.33		
L5	150	75	75	8	387564.55	[19]	390650.65	0.80	409633.80	12249.50	5.69	**7**	387722.43
M1	125	100	25	11	398593.00	[23]	401403.60	0.71	409881.66	4859.34	2.83	**10**	399200.80
M2	125	100	25	10	396916.97	[24]	398735.18	0.46	407392.11	5176.06	2.64		
M3	125	100	25	9	375695.41	[24]	377903.87	0.59	384735.92	3672.35	2.41		
M4	125	100	25	7	348140.16	[24]	348698.63	0.16	357860.86	5231.01	2.79		
N1	150	100	50	11	408100.62	[24]	412109.46	0.98	424331.60	6096.19	3.98	**10**	411677.88
N2	150	100	50	10	408065.44	[24]	414899.30	1.67	425156.91	6159.91	4.19		
N3	150	100	50	9	394337.86	[19]	396827.05	0.63	410438.49	5145.72	4.08		
N4	150	100	50	10	394788.37	[24]	400874.44	1.54	410938.56	5871.72	4.09	**9**	400609.02
N5	150	100	50	7	373476.30	[19]	379504.97	1.61	390531.64	7183.68	4.57		
N6	150	100	50	8	373758.65	[19]	381530.98	2.08	389583.94	4587.00	4.23	**7**	381062.82
Average								0.41			2.67		

Table 2: Best-known results compared to the results from SSMOEA on the Salhi and Nagy [25] VRPMB instances.

Instance				Best-known			Lowest cost found by MOEA					
Id.	N	N_L	N_B	k	Cost	Ref.	k	Cost	%	Avg.	Std.	%
CMT1H	50	25	25	3	462	[34]	3	468	1.30	481.42	10.53	4.20
CMT1Q	50	38	12	4	490	[34]	4	490	0.00	504.64	10.53	2.99
CMT1T	50	45	5	5	520	[19]	5	520	0.00	525.53	6.53	1.06
CMT2H	75	38	37	6	661	[34]	6	668	1.06	684.64	12.61	3.58
CMT2Q	75	57	18	8	733	[19]	8	734	0.14	751.52	7.32	2.53
CMT2T	75	68	7	9	783	[19]	9	793	1.28	806.62	10.50	3.02
CMT3H	100	50	50	5	701	[19]	5	717	2.28	734.14	11.36	4.73
CMT3Q	100	75	25	6	747	[19]	6	751	0.54	777.84	11.33	4.13
CMT3T	100	90	10	7	798	[19]	7	806	1.00	827.82	11.31	3.74
CMT4H	150	75	75	7	829	[19]	7	838	1.09	865.00	18.31	4.34
CMT4Q	150	113	37	9	915	[34]	9	921	0.66	945.52	14.76	3.34
CMT4T	150	135	15	11	996	[34]	**11**	**993**	**-0.30**	1018.81	13.67	2.29
CMT6H	50	25	25	6	555	[27]	6	555	0.00	568.40	13.10	2.41
CMT6Q	50	38	12	6	555	[19]	6	555	0.00	565.87	10.02	1.96
CMT6T	50	45	5	6	555	[19]	6	555	0.00	571.17	15.98	2.91
CMT7H	75	38	37	11	900	[19]	11	902	0.22	924.42	15.90	2.71
CMT7Q	75	57	18	11	901	[34]	11	905	0.44	925.02	14.30	2.67
CMT7T	75	68	7	11	903	[19]	11	907	0.44	928.33	14.36	2.81
CMT8H	100	50	50	9	866	[34]	**9**	**865**	**-0.12**	883.08	19.12	1.97
CMT8Q	100	75	25	9	866	[34]	**9**	**865**	**-0.12**	880.32	15.36	1.65
CMT8T	100	90	10	9	866	[34]	9	866	0.00	889.72	14.56	2.74
CMT9H	150	75	75	14	1159	[25]	14	1167	0.69	1195.97	16.19	3.19
CMT9Q	150	113	37	14	1162	[19]	14	1168	0.52	1192.46	15.75	2.62
CMT9T	150	135	15	14	1164	[25]	14	1164	0.00	1196.64	20.34	2.80
CMT11H	120	60	60	4	818	[19]	4	818	0.00	836.87	14.44	2.31
CMT11Q	120	90	30	6	939	[19]	6	941	0.21	950.13	9.94	1.19
CMT11T	120	108	12	7	999	[34]	7	1001	0.20	1015.51	25.63	1.65
CMT12H	100	50	50	5	629	[19]	5	630	0.16	664.60	13.93	5.66
CMT12Q	100	75	25	7	729	[19]	8	740	1.51	764.13	12.02	4.82
CMT12T	100	90	10	9	788	[34]	9	790	0.25	807.77	5.41	2.51
CMT13H	120	60	60	11	1540	[19]	11	1546	0.39	1564.16	10.50	1.57
CMT13Q	120	90	30	11	1543	[19]	11	1546	0.19	1561.22	11.51	1.18
CMT13T	120	108	12	11	1544	[19]	11	1545	0.06	1564.59	15.31	1.33
CMT14H	100	50	50	10	822	[34]	10	822	0.00	840.81	13.61	2.29
CMT14Q	100	75	25	10	822	[34]	10	822	0.00	843.91	17.10	2.67
CMT14T	100	90	10	10	827	[34]	10	828	0.12	851.94	12.92	3.02
Average									0.40			2.79

which means they dominate the best-known. The remaining six solutions do not have a lower cost than the best-known, but they still have a smaller number of routes, so they are not dominated by the best-known if both objectives are considered equally important.

6.1.2. VRPMB

Table 2 presents the details of the VRPMB results. The first four columns show the properties of each benchmark instance, and the next three columns give the number of routes and cost of the best-known solution, and the study where it was first presented. Here, the number of routes is not fixed by the problem instance, but given by the solution with the lowest cost. The following three columns show the number of routes and cost of the best result from SSMOEA and the percentage gap between this and the best-known result. The final three columns show the average of the best result obtained by SSMOEA from each

Table 3: Best-known results compared to the results from SSMOEA on the Salhi and Nagy [25] VRPSDP instances.

Instance		Best-known			Lowest cost found by MOEA					
Id.	N	k	Cost	Ref.	k	Cost	%	Avg.	Std.	%
CMT1X	50	3	466.77	[48]	3	466.75	0.00	485.34	9.13	3.98
CMT1Y	50	3	458.96	[49]	3	472.36	2.92	487.20	7.86	6.15
CMT2X	75	6	668.77	[49]	6	695.69	4.03	713.62	8.21	6.71
CMT2Y	75	6	663.25	[49]	7	695.04	4.79	711.52	8.65	7.28
CMT3X	100	5	715.32	[34]	5	724.80	1.33	745.67	11.91	4.24
CMT3Y	100	5	719.00	[30]	5	728.24	1.29	742.32	10.24	3.24
CMT4X	150	7	852.46	[48]	7	870.75	2.15	899.14	16.32	5.48
CMT4Y	150	7	847.58	[34]	7	868.20	2.43	895.13	12.01	5.61
CMT6X	50	6	555.43	[34]	6	556.63	0.22	570.95	14.97	2.79
CMT6Y	50	6	555.43	[34]	6	555.39	-0.01	571.75	13.05	2.94
CMT7X	75	11	901.00	[19]	11	901.21	0.02	925.44	19.88	2.71
CMT7Y	75	11	901.10	[34]	11	901.21	0.01	923.21	16.84	2.45
CMT8X	100	9	865.50	[34]	9	867.65	0.25	889.32	16.98	2.75
CMT8Y	100	9	865.50	[34]	9	867.65	0.25	887.55	16.08	2.55
CMT9X	150	14	1161.37	[34]	14	1164.61	0.28	1187.66	14.02	2.26
CMT9Y	150	14	1161.37	[34]	14	1163.99	0.23	1191.07	17.2	2.56
CMT11X	120	4	833.92	[48]	4	837.30	0.41	873.76	26.77	4.78
CMT11Y	120	4	830.39	[49]	4	836.59	0.75	872.52	23.33	5.07
CMT12X	100	6	644.70	[49]	6	675.90	4.84	687.39	7.11	6.62
CMT12Y	100	6	659.52	[49]	6	663.80	0.65	686.86	11.43	4.15
CMT13X	120	11	1549.79	[34]	**11**	**1546.30**	**-0.23**	1559.75	13.13	0.64
CMT13Y	120	11	1544.37	[34]	11	1545.69	0.09	1565.78	15.77	1.39
CMT14X	100	10	821.75	[50]	10	821.80	0.01	845.09	19.01	2.84
CMT14Y	100	10	821.75	[34]	10	821.80	0.01	844.99	17.35	2.83
Average							1.11			3.83

of the 30 repetitions, its corresponding standard deviation, and the percentage gap between the average best result and the best-known.

For three instances, SSMOEA found a new best-known solution (shown in bold), and for ten instances it found solutions that equal the existing best-known. For another 16 instances, the average gap between the best-know results and the one found by SSMOEA is less than 1.00%. On average, the best solutions found by SSMOEA were only 0.40% worse than the best-known. It can also be seen that the average gap between the average best result found by SSMOEA and the best-known result is 2.79%.

6.1.3. VRPSDP

Table 3 presents the details of the VRPSDP results, using the same column headings as Table 2. In this case, SSMOEA was able to find solutions that improved upon the current best-known for one instance (shown in bold), and solutions that were equal (within rounding differences) to the best-known for six instances. Results for another nine of the remaining 17 instances are no more than 1.00% above the best-known. On average, the percentage gap between the best solutions found by SSMOEA and the best-known was only 1.11%, and the average gap between the average best result found by SSMOEA and the best-known result is 3.83%.

6.1.4. *Summary of single-objective results*

The purpose of this section was to establish that the individual solutions found by the proposed fully multi-objective algorithm (SSMOEA) are not too much worse than the special-purpose single objective approaches presented previously, despite the disadvantage of working to spread the solutions across the whole Pareto front. For the basic VRPB, the SSMOEA results were on average only 0.41% worse than the best-known, and for many problem instances the best-known results were equaled. Moreover, SSMOEA was able to find solutions that dominate the best-known for 11 instances and have a smaller number of routes for another 6 instances. For the VRPMB, the results were on average only 0.40% worse than the previous best-known, three new best-known solution were found, and for ten problem instances the existing best-known results were equaled. Finally, for the VRPSDP, the results were on average only 1.11% worse than the previous best-known, one new best-known solution was found, and for six problem instances the existing best-known results were equaled. The conclusion is that, while SSMOEA does not always produce the best possible single-objective solutions, it is not far off, and sometimes it manages to produce better solutions than the previous single-objective approaches.

6.2. *Multi-objective performance comparison with NSGA-II and MOEA/D*

One of the main contributions of this paper is the study of fully multi-objective solutions of the VRPB variants, involving the minimization of two or three objective functions. As noted above, the performances of multi-objective optimizers cannot be reliably compared using the kind of simple averages employed in the previous section. For that reason, the results will now be analyzed using the hypervolume performance metric described in Section 4.1. Since no previous studies have presented multi-objective results, the non-dominated solutions from SSMOEA are compared with those obtained using the crowding mechanism of the widely-used optimizer NSGA-II [15] and the decomposition approach of MOEA/D [16] that have proved successful on numerous other multi-objective problems. The differences in the results are then be explored further using the population diversity measure described in Section 5.1.4. For the sake of brevity, only a summary of the multi-objective performances over categories are presented, rather than full solution listings for each problem instance[1].

6.2.1. *Baseline optimizers*

An advantage of selecting NSGA-II for comparing performances is that it can be set up with many similarities to the proposed SSMOEA, allowing a clear test of their novel features. So, to make the comparisons as clear as possible, NSGA-II was implemented using exactly the same solution representation, selection and crossover and mutation operators that were designed for SSMOEA. The key difference is the way in which the parent and survival selection is performed to enhance diversity. NSGA-II follows the standard EA approach of choosing

[1]Detailed results are available on request from the first author.

both parents according to highest fitness. Instead, SSMOEA chooses one parent on the basis of highest fitness, but the other one according to lowest solution similarity. For determining survival, SSMOEA takes solution similarity into account, while NSGA-II employs a crowding distance measure that does not involve any problem specific routing information at all.

MOEA/D was selected because it is one of the current best multi-objective optimizers and has been shown to offer improved performance on two problems closely related to the VRP [51, 52]. It works by explicitly decomposing the multi-objective problem defined in (7), with F objective functions f_i, into M scalar optimization sub-problems j of the form

$$\text{minimize } \phi(\mathbf{x}|\boldsymbol{\lambda}^j) = \sum_{i=1}^{F} \lambda_i^j f_i(\mathbf{x}) \ , \tag{16}$$

where $\mathbf{x}$ is a problem solution, $\phi(\mathbf{x}|\boldsymbol{\lambda}^j)$ is the objective function of sub-problem j, and $\boldsymbol{\lambda}^j = (\lambda_1^j, \ldots, \lambda_F^j)$ is the corresponding weight vector with positive components that sum to 1. Then, the optimal solution to each scalar problem (16) is a Pareto optimal solution of the full problem (7), and using a uniform distribution of M weight vectors $\{\boldsymbol{\lambda}^j\}$ gives a set of M different Pareto optimal solutions. MOEA/D minimizes all M objective functions simultaneously in a single run by maintaining a population composed of the best solution found so far for each subproblem [16]. It also maintains an archive of the non-dominated solutions of the original multi-objective problem found during the search.

Although MOEA/D has little similarity to the operation of SSMOEA, it can be implemented with the same solution representation, crossover and mutation operators. It follows the same sequence of stages as any evolutionary algorithm, except that, after reproduction, the offspring is submitted to an improvement heuristic instead of the mutation stage. If the resulting offspring dominates any solutions in the archive, those solutions are removed from it. If no solution in the archive dominates the offspring, it is added to the archive. Further details can be found in the original publication of Zhang and Li [16].

In order to render the comparisons as fair as possible, the parameter values for NSGA-II were set equal to those of SSMOEA. For MOEA/D, totally fair comparisons were more difficult. In particular: requiring a reasonable spread of M weight vectors and corresponding sub-problems sometimes means allowing a bigger population size for MOEA/D than SSMOEA and NSGA-II; then, because MOEA/D maintains a solution archive, it has another advantage over SSMOEA and NSGA-II [51]; and, finally, the usual local search procedure of MOEA/D has to be replaced by the same mutation stage as SSMOEA and NSGA-II, to avoid another unfair advantage [52].

6.2.2. Minimization of the number of routes and travel cost

Tables 1, 2 and 3 have already presented the best results for SSMOEA bi-objective optimization, taking into account the lowest travel cost only. Now the bi-objective performance needs to be studied, i.e. the full Pareto approximation resulting from minimizing both the travel cost and number of routes.

Table 4: Significant differences of the hypervolume $\mathcal{H}$ and diversity $\mathcal{D}$ metrics applied to the non-dominated sets found by SSMOEA (SS), NSGA-II (NS) and MOEAD/D (MD) while solving the bi-objective (number of routes and travel cost) VRPB, VRPMB, and VRPSDP.

Category		Hypervolume ($\mathcal{H}$)				Diversity ($\mathcal{D}$)			
Id.	#I	SS>NS	SS<NS	SS>MD	SS<MD	SS>NS	SS<NS	SS>MD	SS<MD
				VRPB					
A	4	0	0	3	0	0	0	0	1
B	3	0	0	3	0	0	0	0	0
C	4	0	0	2	0	0	1	0	0
D	4	0	0	2	0	0	0	4	0
E	3	2	0	2	0	0	0	3	0
F	4	0	0	1	0	0	0	0	0
G	6	0	0	1	0	0	0	6	0
H	6	0	0	1	0	0	0	5	0
I	5	0	0	1	0	1	0	5	0
J	4	2	0	0	0	0	0	4	0
K	4	1	0	1	0	0	0	0	0
L	5	0	0	0	2	1	0	5	0
M	4	0	0	0	1	0	0	4	0
N	6	0	0	0	3	1	0	1	1
				VRPMB					
H	12	2	0	5	3	4	0	4	3
Q	12	3	0	3	2	3	0	5	4
T	12	1	0	3	2	1	0	3	3
				VRPSDP					
X	12	4	0	7	3	1	0	5	3
Y	12	2	0	7	3	2	0	5	3

As noted in Section 4.1, to compute the hypervolumes covered by the non-dominated sets, an appropriate reference point is required that has maximal objective function values. For each VRPB instance, there will be a suitable reference solution (that will not necessarily be a real feasible solution) formed by N routes, with one customer allocated to each route. That solution clearly has the maximal number of routes N, and a maximal travel cost C_{max} which is twice the cost from the depot to every customer. Thus, the reference point is $\mathbf{z} = (N, C_{max})$, and the greater the hypervolume defined by that point and the Pareto approximation set, the better the solution.

For each problem instance and SSMOEA, NSGA-II or MOEA/D repetition, the non-dominated solution set was taken and the covered hypervolume computed. Then, a *t-test* (two-sample, two-tailed, unequal variance) was applied to the two pairs of vectors of 30 hypervolume values, from SSMOEA and NSGA-II, and SSMOEA and MOEA/D, respectively, to determine whether the difference in average hypervolume was statistically significant at the 95% confidence level. For each Pareto approximation, the diversity of the non-dominated solutions was also computed, using equation (15), and a *t-test* used (in the same way as for the hypervolume) to determine whether any diversity differences were significant.

The hypervolume $\mathcal{H}$ and diversity $\mathcal{D}$ results for the basic VRPB, VRPMB, and VRPSDP, grouped by instance category, are shown in Table 4. The first two columns show the instance category identifier and number of instances within the category. The remaining eight columns are divided into two groups: the first four columns correspond to the hypervolume metric and the last four to the diversity metric. In the case of the hypervolume, the first two columns, titled SS>NS and SS<NS, represent the number of instances for which solutions from SSMOEA (SS) delimit a significantly larger hypervolume than solutions from NSGA-II (NS), and viceversa. The following two columns represent the same comparison between SSMOEA and MOEA/D (MD). The last four columns show corresponding statistical significance numbers for the diversity metric.

In the case of the VRPB, the difference in hypervolume covered by the non-dominated solutions from SSMOEA and NSGA-II is significant in only five instances, which all have SSMOEA better than NSGA-II. The hypervolumes for the remaining 57 instances have no statistically significant differences. There are 23 significant differences between the hypervolumes covered by SSMOEA and MOEA/D, with SSMOEA better in 17 instances, and MOEA/D better in 6. For the remaining 39 instances, there are no significant differences. Regarding population diversity, there is a significant difference between the solutions found by SSMOEA and NSGA-II in four instances, in three of which SSMOEA is higher, and one for which NSGA-II is higher. Again there are more significant differences between SSMOEA and MOEA/D, with SSMOEA having higher diversity in 37 instances, and MOEA/D higher in only 2.

For the VRPMB, the SSMOEA solution hypervolumes are significantly larger than those for NSGA-II in 6 instances, while NSGA-II solutions have no significantly larger hypervolumes. In the remaining 30 instances, there are no significant hypervolume differences. Again there are more significant hypervolume differences between SSMOEA and MOEA/D, with SSMOEA better in 11 instances and MOEA/D better in 7. With respect to solution diversity, the SSMOEA solutions are significantly more diverse than those from NSGA-II in 8 instances, and the remaining instances exhibit no significant differences. Solutions from SSMOEA are significantly more diverse that those from MOEA/D in 12 instances, while solutions from MOEA/D are significantly more diverse that those from SSMOEA in 10 instances.

For the VRPSDP, the hypervolumes for SSMOEA solutions are significantly larger than those from NSGA-II in 6 instances, and the remaining 18 instances exhibit no significant differences. Hypervolumes from SSMOEA are significantly better than those from MOEA/D in 14 instances, and MOEA/D is significantly better than SSMOEA in 6. Regarding solution diversity, SSMOEA gives significantly more diversity than NSGA-II for 3 instances, and there are no significant differences for the remaining 21 instances. SSMOEA solution diversity is significantly higher than that from MOEA/D in 10 instances, while MOEA/D solution diversity is significantly higher in 6 instances.

Three more important performance indicators, other than hypervolume and diversity, are the average Pareto approximation sizes, numbers of generations, and execution times. These are presented in Table 5, with the first column showing the instance category, and three sub-columns corresponding to SSMOEA (SS), NSGA-II (NS) and MOEA/D (MD) for each indicator.

Table 5: Averages of the Pareto approximation sizes, the number of generations, and the execution times of SSMOEA (SS), NSGA-II (NS) and MOEA/D (MD) while solving the bi-objective (number of routes and travel cost) VRPB, VRPMB, and VRPSDP.

Cat.	Pareto approx. size			Number of generations			Execution time (s)		
Id.	SS	NS	MD	SS	NS	MD	SS	NS	MD
					VRPB				
A	1.28	1.26	1.45	350.83	362.92	2500.00	0.81	0.80	4.70
B	1.20	1.19	1.31	362.22	379.44	3000.00	1.33	1.37	8.57
C	1.45	1.47	1.36	489.17	503.33	4000.00	3.93	3.93	23.30
D	1.48	1.50	1.48	733.34	675.42	3800.00	5.13	4.53	18.23
E	1.04	1.01	1.01	767.78	796.67	4500.00	7.74	8.90	32.85
F	1.46	1.52	1.55	852.92	897.92	6000.00	23.24	22.78	102.51
G	1.19	1.13	1.31	1197.22	1221.11	5700.00	23.12	22.92	77.16
H	1.22	1.25	1.34	1392.78	1376.39	6800.00	47.22	46.90	160.12
I	1.09	1.05	1.15	1498.33	1576.67	9000.00	123.98	158.93	534.22
J	1.03	1.05	1.11	3388.75	3365.84	9400.00	271.26	256.37	548.59
K	1.02	1.00	1.14	3660.42	3710.83	11300.00	544.89	526.78	1192.94
L	1.39	1.37	1.45	4005.67	4141.33	15000.00	1945.26	1811.58	4131.17
M	1.03	1.04	1.34	5123.75	5186.67	12500.00	997.98	1118.04	1840.45
N	1.01	1.02	1.08	5884.72	6071.95	15000.00	2265.13	2420.08	4044.08
					VRPMB				
H	1.01	1.01	1.32	2056.39	1653.75	9916.67	487.19	404.75	1654.23
Q	1.02	1.01	1.37	2246.11	1813.75	9916.67	526.45	413.02	1481.95
T	1.07	1.07	1.46	2498.75	1987.78	9916.67	550.21	424.67	1457.67
					VRPSDP				
X	1.22	1.26	1.92	2416.67	2010.56	9916.67	580.47	476.09	1709.74
Y	1.23	1.26	1.84	2455.97	2037.08	9916.67	596.35	487.36	1773.63

One aspect of the VRPB variants that was not clear from the earlier single-objective studies is the extent to which they really are multi-objective problems. In other words, are the multiple objectives really in conflict, or could a single solution simultaneously optimize all the objectives? Table 5 shows that the average number of solutions per instance is not much more than one for all three algorithms, suggesting that the two objectives are actually rarely in conflict. This is far fewer solutions than for other VRP benchmarks, such as those for VRPTW [14], and may explain why there is relatively little difference between SSMOEA, NSGA-II, and MOEA/D on these problems. This is probably because the instances were really designed for the single objective problem.

Table 5 also shows that SSMOEA runs for slightly fewer generations than NSGA-II in 11 out of the 14 categories, but the average execution time was shorter in only 5 categories. On average, both SSMOEA and NSGA-II executed similar numbers of generations in the same time. Surprisingly, MOEA/D always ran for the maximum number of generations, even though that was set to many times that needed by the other algorithms, leading to longer execution times.

In the cases of the VRPMB and VRPSDP, the size of the Pareto approximations is again very small, with rarely more than one solution found for each problem instance. In these cases, SSMOEA runs for approximately 25% more generations than NSGA-II, with similarly longer execution times. This is to be expected given SSMOEA's more sophisticated diversity preservation procedures. Again, MOEA/D ran for the maximum number of generations, giving the longest execution times.

Overall then, for optimizing the number of routes and travel cost for the VRPB, VRPMB and VRPSDP, the proposed SSMOEA offers only a slight improvement over NSGA-II and MOEA/D, with the vast majority of benchmark instances showing no significant difference between the hypervolumes and diversities of the non-dominated solutions found by them. The obvious conjecture is that this similar performance is due to the benchmark instances being designed for the single-objective problems, and consequently, as suggested by Table 5, very few of them have conflicting objectives. There is a clear need for better benchmark instances, with more conflicts between the objectives, to test the differences more conclusively.

6.2.3. Minimization of the travel cost and uncollected profit

The performance of the algorithms on minimizing a different pair of objectives, namely travel cost and uncollected profit, is now analyzed for the VRPSB and VRPMSB. This is interesting because, to minimize the uncollected profit, more backhaul customers have to be visited, which will increase the travel cost. And to minimize the travel cost, fewer backhaul customer can be serviced, which will increase the uncollected profit. This should provide the conflict in objectives needed to demonstrate the advantages of the proposed SSMOEA.

Since the benchmark instances used in the previous studies have not included profit information for the backhaul customers, this needed to be introduced. For simplicity, each backhaul customer supply s_i was taken to provide the same profit p_i. Then, for each problem instance, there will be a maximal profit $P_{max} = \sum_{i \in \mathcal{V}_B} p_i$ that could be collected. Hence, the natural reference point for computing the hypervolume performance metric is $\mathbf{z} = (C_{max}, P_{max})$.

For each run of each algorithm, the non-dominated set was recorded and the hypervolume and diversity computed. Then, for each problem instance, those performance measures were submitted to a *t-test* as previously. The results for the VRPSB and VRPMSB are presented in Table 6, which has the same structure as Table 4. Here, the VRPSDP was not considered because all customers must be visited which results in zero uncollected demand for all cases.

For the VRPSB, the difference in hypervolume covered by the non-dominated solutions from SSMOEA and NSGA-II is significant in 36 of the 62 instances, with the SSMOEA solutions defining a larger hypervolume than NSGA-II in all those cases. Moreover, solutions from SSMOEA delimit a significantly larger hypervolume than those from MOEA/D in all 62 instances. Regarding population diversity, there is a significant difference between SSMOEA and NSGA-II in all but three of the instances, with the solution diversity higher for SSMOEA in all those cases. Moreover, solutions from SSMOEA are significantly more diverse than those from MOEA/D in all 62 instances. As intended, the higher SSMOEA diversities are correlated with the higher hypervolumes.

For the VRPMSB, the hypervolumes defined by the SSMOEA solutions are significantly larger than those of the NSGA-II solutions in 21 of the 36 instances, and significantly larger than those of the MOEA/D solutions in 25 instances,

Table 6: Significant differences of the hypervolume $\mathcal{H}$ and diversity $\mathcal{D}$ metrics applied to the non-dominated sets found by SSMOEA (SS), NSGA-II (NS) and MOEA/D (MD) while solving the bi-objective (travel cost and uncollected profit) VRPSB and VRPMSB.

Category		Hypervolume ($\mathcal{H}$)				Diversity ($\mathcal{D}$)			
Id.	#I	SS>NS	SS<NS	SS>MD	SS<MD	SS>NS	SS<NS	SS>MD	SS<MD
				VRPSB					
A	4	0	0	4	0	3	0	4	0
B	3	0	0	3	0	1	0	3	0
C	4	4	0	4	0	4	0	4	0
D	4	0	0	4	0	4	0	4	0
E	3	3	0	3	0	3	0	3	0
F	4	4	0	4	0	4	0	4	0
G	6	1	0	6	0	6	0	6	0
H	6	6	0	6	0	6	0	6	0
I	5	5	0	5	0	5	0	5	0
J	4	0	0	4	0	4	0	4	0
K	4	4	0	4	0	4	0	4	0
L	5	5	0	5	0	5	0	5	0
M	4	0	0	4	0	4	0	4	0
N	6	4	0	6	0	6	0	6	0
				VRPMSB					
H	12	11	0	9	1	12	0	12	0
Q	12	5	0	8	0	12	0	12	0
T	12	5	0	8	0	11	0	12	0

while solutions from NSGA-II have no significantly larger hypervolumes, and solutions from MOEA/D have a significantly larger hypervolume in only one instance. Regarding solution diversity, SSMOEA solutions are significantly more diverse than those from NSGA-II for all but one instance, and significantly more diverse than those from MOEA/D for all 36 instances.

Table 7 presents the corresponding Pareto approximation sizes, numbers of generations and execution times, with the same format as that of Table 5.

For the VRPSB, the sizes of the Pareto approximations for SSMOEA and NSGA-II are similar, while those for MOEA/D are much smaller. However, all these sizes are much larger than when minimizing the number of routes and travel cost, with the non-dominated sets here containing from 8 to 150 solutions on average. The downside of the improved SSMOEA performance is that it runs on average for approximately 13% and 83% more generations than NSGA-II and MOEA/D, with nearly 39% and 300% longer execution times. Here, MOEA/D requires fewer generations than SSMOEA and NSGA-II.

The sizes of the non-dominated sets for the VRPMSB are all large, and similar for all three approaches. Here, SSMOEA requires, on average, about 28% and 73% more generations than NSGA-II and MOEA/D, and its execution times are approximately 37% and 157% longer.

To help visualize how the above results relate to the distribution of non-dominated solutions in the objective space, Figure 1 shows the Pareto approximations from one typical run of SSMOEA, NSGA-II and MOEA/D while solving the bi-objective problems VRPSB instance F1 and VRPMSB instance 7Q. In both cases, it can be seen how the non-dominated solutions from SSMOEA are

Table 7: Averages of the Pareto approximation sizes, the number of generations, and the execution times of SSMOEA (SS), NSGA-II (NS) and MOEA/D (MD) while solving the bi-objective (travel cost and uncollected profit) VRPSB and VRPMSB.

Cat.	Pareto approx. size			Number of generations			Execution time (s)		
Id.	SS	NS	MD	SS	NS	MD	SS	NS	MD
				VRPSB					
A	10.99	10.85	8.12	651.67	677.08	452.92	2.34	2.36	0.99
B	26.91	26.56	12.99	1010.55	866.11	731.67	6.53	5.87	2.41
C	40.00	39.75	24.22	3333.75	1916.25	1684.17	51.21	26.41	11.92
D	19.39	19.42	11.22	1592.92	1623.75	958.75	14.93	13.61	6.25
E	44.69	44.36	19.88	3083.89	2192.78	1548.33	106.16	38.42	15.44
F	59.98	59.95	32.80	4965.00	4681.25	2745.00	246.78	217.58	56.27
G	42.26	42.72	14.42	3279.17	2914.72	1585.00	109.09	85.56	29.32
H	67.96	67.85	24.48	6189.17	4720.28	2743.89	425.13	289.84	71.14
I	89.99	87.77	57.52	8868.00	9022.67	5762.00	1637.65	1122.47	482.04
J	53.91	57.32	18.86	8343.75	7427.50	3509.17	1050.30	821.11	264.94
K	112.85	110.98	33.21	11048.75	11017.92	6548.34	3693.46	2620.77	808.90
L	149.77	147.85	70.53	15000.00	15000.00	10152.67	12547.58	9196.34	4064.85
M	97.18	100.78	27.77	12208.33	12308.33	5182.08	4510.26	4763.83	955.19
N	149.38	147.67	44.79	14894.72	14949.17	8964.17	10921.84	8903.66	3177.56
				VRPMSB					
H	93.19	86.37	109.06	9843.61	9426.94	8417.92	5401.78	4079.55	4122.87
Q	71.89	67.96	53.93	9538.75	8004.86	5623.64	4434.93	3638.27	1379.00
T	23.64	23.34	19.62	9096.67	5666.53	3944.86	3625.51	2332.19	1139.31

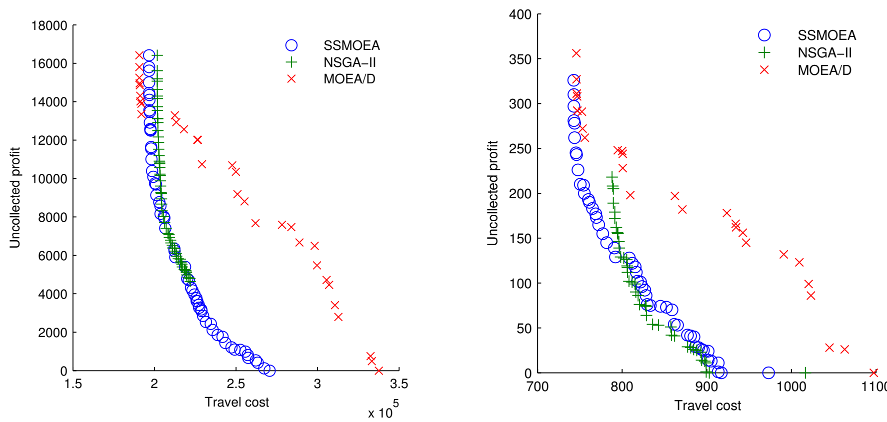
more uniformly distributed and cover a wider objective space than the Pareto approximations from NSGA-II and MOEA/D.

6.2.4. Minimization of the number of routes, travel cost and uncollected profit

The three algorithms were also tested on the tri-objective VRPSB and VRPMSB, minimizing the number of routes, total travel cost, and uncollected profit. In this case, the reference point for computing the hypervolume performance metric is $\mathbf{z} = (N, C_{max}, P_{max})$. As before, the non-dominated set was recorded for each run of each algorithm, with the hypervolumes and diversities computed and subjected to *t-tests*. The results for the VRPSB and VRPMSB are presented in Table 8, with the same format as Table 4.

For the VRPSB, the solutions produced by SSMOEA delimit hypervolumes that are significantly better than NSGA-II for 38 problem instances and better than MOEA/D for all 62 instances, while NSGA-II solutions have better hypervolume in only one instance. Regarding solution diversity, SSMOEA solutions have significantly higher diversity than those from NSGA-II in 60 of the 62 instances, and in 61 instances compared with solutions from MOEA/D.

For the VRPMSB, the hypervolume covered by SSMOEA solutions is significantly better than that covered by the NSGA-II solutions in 18 instances, and better than that covered by the MOEA/D solutions in 34 instances. Both NSGA-II and MOEA/D are better in none. Furthermore, the solution diversity from SSMOEA is significantly better than that from NSGA-II in 33 of the 36 instances, and better than that from MOEA/D in 31 instances. NSGA-II is better in none and MOEA/D is better in three instances.



(a) (b)

Figure 1: Pareto approximations from one run of SSMOEA, NSGA-II, and MOEA/D for the bi-objective problems: (a) VRPSB instance F1, and (b) VRPMSB instance 7Q.

Table 8: Significant differences of the hypervolume $\mathcal{H}$ and diversity $\mathcal{D}$ metrics for the non-dominated sets found by SSMOEA (SS), NSGA-II (NS) and MOEA/D (MD) while solving the tri-objective (number of routes, travel cost and uncollected profit) VRPSB and VRPMSB.

Category		Hypervolume ($\mathcal{H}$)				Diversity ($\mathcal{D}$)			
Id.	#I	SS>NS	SS<NS	SS>MD	SS<MD	SS>NS	SS<NS	SS>MD	SS<MD
					VRPSB				
A	4	1	0	4	0	2	0	4	0
B	3	1	0	3	0	3	0	2	0
C	4	4	0	4	0	4	0	4	0
D	4	2	0	4	0	4	0	4	0
E	3	2	0	3	0	3	0	3	0
F	4	4	0	4	0	4	0	4	0
G	6	2	0	6	0	6	0	6	0
H	6	6	0	6	0	6	0	6	0
I	5	5	0	5	0	5	0	5	0
J	4	0	0	4	0	4	0	4	0
K	4	4	0	4	0	4	0	4	0
L	5	4	0	5	0	5	0	5	0
M	4	0	1	4	0	4	0	4	0
N	6	3	0	6	0	6	0	6	0
					VRPMSB				
H	12	9	0	12	0	12	0	12	0
Q	12	5	0	10	0	12	0	12	0
T	12	4	0	12	0	9	0	7	3

Table 9 presents the corresponding Pareto approximation sizes, numbers of generations and execution times, in the same format as Table 5.

For the VRPSB, the sizes of the Pareto approximations from SSMOEA and NSGA-II are again large and similar, while those from MOEA/D are slightly smaller. The number of generations SSMOEA and NSGA-II run, on average, have a more variable behavior. For some categories, SSMOEA runs for as many as 21% more generations than NSGA-II, e.g. categories B and J, while for

Table 9: Averages of the Pareto approximation sizes, the number of generations, and the execution times of SSMOEA (SS), NSGA-II (NS) and MOEA/D (MD) while solving the tri-objective (number of routes, travel cost and uncollected profit) VRPSB and VRPMSB.

Cat.	Pareto approx. size			Number of generations			Execution time (s)		
Id.	SS	NS	MD	SS	NS	MD	SS	NS	MD
				VRPSB					
A	14.01	13.13	10.88	1578.75	1516.25	942.92	6.35	5.03	2.66
B	28.13	27.94	16.31	2042.78	1864.44	1410.00	14.56	9.50	6.75
C	39.98	39.27	35.33	5794.17	5567.92	3502.50	87.58	63.22	35.38
D	27.96	22.82	16.62	2703.34	2542.09	1965.00	43.16	21.53	17.19
E	44.78	44.00	28.34	4688.33	6117.22	3665.00	87.25	98.31	39.16
F	59.98	59.69	40.84	7911.67	9050.00	4840.83	411.58	319.70	125.48
G	45.05	43.52	19.97	5265.28	5149.17	3428.61	175.64	155.19	84.85
H	67.94	67.74	33.50	9239.17	10178.06	6013.06	748.21	611.22	214.57
I	89.97	88.95	62.60	13470.67	13500.00	8906.33	2453.90	2242.27	716.27
J	63.58	58.64	33.34	11624.58	9619.17	8150.83	1560.06	1088.47	748.49
K	112.91	112.42	44.05	16303.34	16965.84	11797.09	4391.49	4015.06	1699.17
L	149.75	148.46	86.85	22455.00	22500.00	18336.00	19744.19	14548.04	7253.42
M	102.57	104.71	43.24	17781.67	17187.08	12734.17	6172.67	5145.01	2879.71
N	149.44	148.87	53.67	22316.67	22500.00	15320.28	16035.08	13223.83	5824.61
				VRPMSB					
H	96.11	95.98	81.11	14518.33	14070.70	10394.17	7944.01	6608.98	4891.42
Q	75.62	73.76	40.29	13945.97	12520.14	6883.75	7012.49	5749.64	3141.77
T	23.91	23.63	15.88	13367.22	8632.36	4097.78	5626.32	3554.35	1403.67

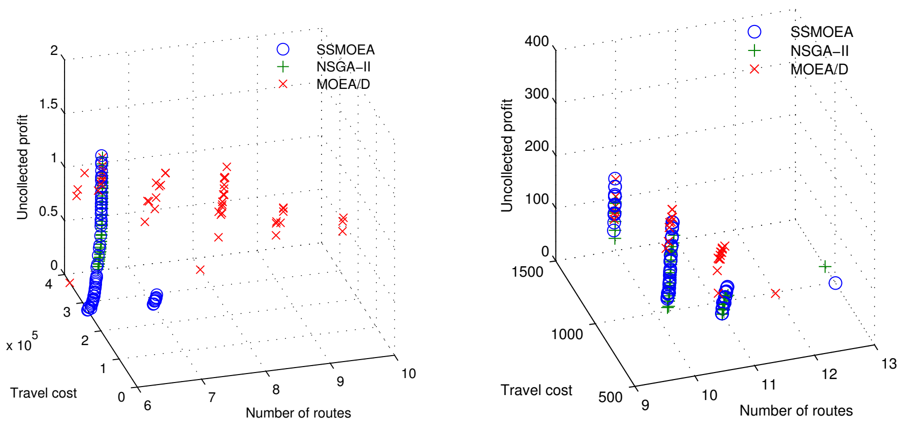
others, NSGA-II requires as many as 23% more generations than SSMOEA, e.g. categories E and F. Considering the average over all categories, both algorithms require similar numbers of generations, but SSMOEA's average execution time is about 30% longer than NSGA-II. MOEA/D again runs for fewer generations with shorter execution times than both SSMOEA and NSGA-II.

Regarding the VRPMSB, the average sizes of the Pareto approximations found by SSMOEA, NSGA-II, and MOEA/D are again large and similar. The number of generations performed by SSMOEA is, on average, 23% higher than that of NSGA-II and 123% higher than MOEA/D, which correspond to increases of approximately 33% and 162% in the execution times.

To visualize how the non-dominated solutions are distributed in the objective space here, Figure 2 shows the Pareto approximations from a typical run of SSMOEA, NSGA-II, and MOEA/D for the tri-objective VRPSB instance F1 and VRPMSB instance 7Q. In the case of the VRPSB instance F1, although the MOEA/D Pareto approximation spans a wider range for the number of routes, they correspond to a higher travel cost and uncollected profit than the SSMOEA solutions. Meanwhile, the non-dominated solutions from NSGA-II cover a smaller range on the uncollected profit objective than those from SS-MOEA, and they are located on only one value of the number of routes objective. A similar pattern of results is seen for the VRPMSB instance 7Q.

6.2.5. Discovering extreme solutions

A common problem with multi-objective optimizers is that they find a lot of compromise solutions, but can miss the extreme solutions that fully optimize one objective at the expense of the other. Since the minimum uncollected profit



(a) (b)

Figure 2: Pareto approximations from one run of SSMOEA, NSGA-II, and MOEA/D for the tri-objective problems: (a) VRPSB instance F1, and (b) VRPMSB instance 7Q.

is known to be zero, corresponding to the case when all backhaul customers are visited, it is possible to check the extent to which those solutions are being found by SSMOEA, NSGA-II, and MOEA/D. Table 10 presents the percentage of the 30 repetitions, averaged over instance category, that each algorithm was able to find at least one solution with zero uncollected profit. This shows that SSMOEA is much better than NSGA-II at finding the solutions which service all the backhaul customers, since the averages are closer to 100% and the standard deviations are very low. MOEA/D presents a very similar performance to SSMOEA, with a slightly better performance overall.

6.2.6. Summary of multi-objective performance

When SSMOEA, NSGA-II and MOEA/D were set to minimize the number of routes and travel cost, all three approaches performed similarly. There was a slight advantage of SSMOEA over NSGA-II and MOEA/D for a few benchmark instances, but for the vast majority of instances there was no significant difference in the hypervolume or diversity metrics. This is probably because there were few conflicts between objectives that required a multi-objective approach. When the algorithms were set to minimize the travel cost and uncollected profit, there were many conflicts, and SSMOEA solutions delimited significantly larger hypervolumes than those found by NSGA-II and MOEA/D in more than half of the problem instances, for both VRPSB and VRPMSB, and it was also considerably more likely than NSGA-II to find the extreme solutions with no uncollected profit. This situation was repeated when the algorithms were set to minimize all three objectives simultaneously. Overall then, it has been demonstrated that, in cases with conflicting objectives, the proposed diversity enhancing SSMOEA really can provide significantly better multi-objective VRPB solutions than com-

Table 10: Percentage of zero-uncollected-profit solutions found in SSMOEA (SS), NSGA-II (NS) and MOEA/D (MD) Pareto approximations for the bi-objective and tri-objective VRPSB and VRPMSB.

Category	Bi-objective problem			Tri-objective problem		
Id.	SS	NS	MD	SS	NS	MD
			VRPSB			
A	100.00	100.00	100.00	100.00	100.00	100.00
B	98.89	100.00	100.00	100.00	75.56	100.00
C	100.00	0.00	100.00	100.00	2.50	100.00
D	100.00	100.00	100.00	100.00	100.00	100.00
E	100.00	3.33	100.00	100.00	32.22	100.00
F	96.67	0.00	99.17	100.00	1.67	100.00
G	100.00	100.00	100.00	100.00	98.89	100.00
H	100.00	0.00	100.00	100.00	11.11	100.00
I	100.00	0.00	100.00	100.00	0.00	100.00
J	100.00	100.00	100.00	100.00	98.33	100.00
K	98.33	0.00	100.00	97.50	3.33	100.00
L	99.33	0.00	100.00	100.00	12.67	100.00
M	100.00	95.83	100.00	100.00	98.33	100.00
N	100.00	1.11	100.00	100.00	27.22	100.00
Average	99.57	40.32	99.95	99.84	45.59	100.00
Std. Dev.	1.28	48.93	0.42	0.94	44.27	0.00
			VRPMSB			
H	100.00	74.72	99.72	100.00	72.22	100.00
Q	100.00	99.72	99.72	100.00	99.29	100.00
T	100.00	100.00	100.00	92.31	94.15	100.00
Average	100.00	91.48	99.81	97.44	88.55	100.00
Std. Dev.	0.00	14.51	0.16	4.44	14.37	0.00

peting approaches, though that does involve increased execution times of the order of 30% with respect to NSGA-II, and 200% with respect to MOEA/D.

6.3. Performance analysis for different optimization settings

Another important contribution of this paper is analyzing how well the non-dominated sets generated by SSMOEA for the tri-objective problem provide solutions that form good non-dominated sets for the bi-objective sub-problems too. To this end, for each tri-objective instance and repetition, the hypervolume was determined for only the objectives of the relevant bi-objective problem, and *t-tests* applied in the usual way. Also, for each instance, the overall Pareto approximations from all 30 repetitions was obtained and analyzed to see whether they covered or dominated each other. To avoid confusion, the acronym $\mathrm{SSMOEA_{RC}}$ is used when SSMOEA is minimizing the number of routes and travel cost, $\mathrm{SSMOEA_{CP}}$ when it is minimizing the travel cost and uncollected profit, and $\mathrm{SSMOEA_{RCP}}$ when it is minimizing all three objectives.

6.3.1. VRPB and bi-objective VRPSB versus tri-objective VRPSB

From the Pareto approximations found by $\mathrm{SSMOEA_{RCP}}$, two non-dominated sets were extracted for each VRPB instance and repetition: one considering the number of routes and travel cost, and the other considering the travel cost and uncollected profit. Then, the hypervolume metric was computed for each of these non-dominated sets and compared with those found by $\mathrm{SSMOEA_{RC}}$

Table 11: Performance of SSMOEA while solving the VRPB, VRPMB, and bi-objective and tri-objective VRPSB and VRPMSB.

Category		Number of routes and travel cost				Travel cost and uncollected profit			
Id.	#I	$\mathcal{H}$	$\mathcal{H}$(%)	$\preceq$ (%)	$\prec$ (%)	$\mathcal{H}$	$\mathcal{H}$(%)	$\preceq$ (%)	$\prec$ (%)
		VRPB				VRPSB			
A	4	0	0.00	100.00	0.00	1	25.00	98.21	8.12
B	3	0	0.00	100.00	0.00	1	33.33	100.00	1.33
C	4	1	25.00	87.50	0.00	3	75.00	69.30	33.22
D	4	1	25.00	62.50	37.50	3	75.00	100.00	8.22
E	3	0	0.00	33.33	0.00	3	100.00	85.16	61.39
F	4	1	25.00	87.50	37.50	4	100.00	72.76	34.31
G	6	1	16.67	75.00	25.00	6	100.00	97.49	47.27
H	6	0	0.00	16.67	16.67	6	100.00	90.82	80.53
I	5	2	40.00	40.00	40.00	5	100.00	82.39	79.75
J	4	0	0.00	25.00	25.00	4	100.00	86.01	84.76
K	4	0	0.00	25.00	25.00	4	100.00	79.31	79.09
L	5	1	20.00	40.00	40.00	5	100.00	80.49	80.49
M	4	0	0.00	0.00	0.00	4	100.00	64.35	64.35
N	6	0	0.00	16.67	16.67	6	100.00	88.49	88.49
Average			10.83	50.65	18.81		86.31	85.34	53.67
		VRPMB				VRPMSB			
H	12	12	100.00	66.67	58.33	4	33.33	65.59	44.82
Q	12	12	100.00	91.67	58.33	6	50.00	81.46	47.83
T	12	11	91.67	83.33	66.67	3	25.00	72.63	33.76
Average			97.22	80.56	61.11		36.11	73.23	42.14

and $\mathrm{SSMOEA_{CP}}$. Furthermore, from the overall Pareto approximation found by $\mathrm{SSMOEA_{RCP}}$ for each instance from the 30 repetitions, two overall non-dominated sets were extracted: one considering the number of routes and travel cost and the other considering the travel cost and uncollected profit.

The results are presented in Table 11 which has the following structure: The first two columns show the benchmark category and the number of instances in each category. The remaining eight columns are divided into two groups, the first four showing the comparison of the non-dominated sets which consider the number of routes and travel cost, and the last four showing the comparison of the non-dominated sets which consider the travel cost and uncollected profit. The first two columns in each group show the number of instances in each category for which $\mathrm{SSMOEA_{RCP}}$ found solutions that delimit a significantly larger hypervolume than those found by $\mathrm{SSMOEA_{RC}}$ and $\mathrm{SSMOEA_{CP}}$, respectively, and the corresponding percentage. The third column is the percentage of the overall non-dominated set found by $\mathrm{SSMOEA_{RC}}$ and $\mathrm{SSMOEA_{CP}}$, respectively, that is covered ($\preceq$) by the extracted overall non-dominated set found by $\mathrm{SSMOEA_{RCP}}$. The last column is the percentage of the overall non-dominated set found by $\mathrm{SSMOEA_{RC}}$ and $\mathrm{SSMOEA_{CP}}$, respectively, that is dominated ($\prec$) by the extracted overall non-dominated set found by $\mathrm{SSMOEA_{RCP}}$.

In the case of minimizing the number of routes and travel cost, the non-dominated sets found by $\mathrm{SSMOEA_{RCP}}$ delimit a significantly larger hypervolume for 7 of the 62 VRPB instances, and there is no significant difference for the remaining 55 instances. Crucially, the hypervolume defined by non-dominated sets found by $\mathrm{SSMOEA_{RC}}$ is not significantly larger for any problem instance. This means $\text{SSMOEA}_{\text{RCP}}$ is always able to find solutions that define the same, or larger, hypervolume than that delimited by the solutions from $\text{SSMOEA}_{\text{RC}}$. Regarding the overall Pareto approximations, the non-dominated solutions found by $\text{SSMOEA}_{\text{RCP}}$ cover, on average, approximately 51% of those found by $\text{SSMOEA}_{\text{RC}}$, and dominate nearly 19% of them.

For the case when the travel cost and uncollected profit are minimized, the Pareto approximations found by $\text{SSMOEA}_{\text{RCP}}$ define a significantly larger hypervolume than those found by $\text{SSMOEA}_{\text{CP}}$ in 55 of the 62 instances, and for the remaining 7 instances there is no significant difference. This implies that $\text{SSMOEA}_{\text{RCP}}$ is always capable of finding non-dominated solutions that delimit an equal or significantly larger hypervolume than that defined by the non-dominated solutions from $\text{SSMOEA}_{\text{CP}}$. In this case, the overall non-dominated solutions found by $\text{SSMOEA}_{\text{RCP}}$ cover nearly 85% of those found by $\text{SSMOEA}_{\text{CP}}$, and dominate approximately 54% of them.

6.3.2. VRPMB and bi-objective VRPMSB versus tri-objective VRPMSB

For the VRPMSB, exactly the same analysis procedure is followed as for the VRPSB previously, with the results again shown in Table 11. For the case of minimizing the number of routes and travel cost, the Pareto approximations from $\text{SSMOEA}_{\text{RCP}}$ define a significantly larger hypervolume than those from $\text{SSMOEA}_{\text{RC}}$ in 35 of the 36 problem instances. The overall non-dominated sets from $\text{SSMOEA}_{\text{RCP}}$ cover roughly 80% of those from $\text{SSMOEA}_{\text{RC}}$, on average, and dominate about 61% of them.

Regarding the minimization of travel cost and uncollected profit, the Pareto approximations from $\text{SSMOEA}_{\text{RCP}}$ delimit a significantly larger hypervolume than those from $\text{SSMOEA}_{\text{CP}}$ for 13 of the 36 problem instances, and for the remaining 23 instances there is no significant difference. The overall non-dominated solutions from $\text{SSMOEA}_{\text{RCP}}$ cover nearly 73% of those found by $\text{SSMOEA}_{\text{CP}}$, and dominate about 42% of them.

6.3.3. Summary of sub-problem performance analysis

The performance analysis here was designed to determine whether the proposed SSMOEA is still able to find as many non-dominated solutions when it is set to more challenging optimization settings. In fact, it has been demonstrated that, when SSMOEA is set to minimize all three objectives (number of routes, travel cost and uncollected profit), it is capable of finding non-dominated solutions that delimit an equal or larger hypervolume to those defined by the solutions found when it is set to minimize only two objectives. Furthermore, the overall Pareto approximations found when it is set to minimize all three objectives cover at least 50%, and dominate no less than 40%, of those found when it is set to minimize only two objectives.

7. Conclusions

This paper has proposed a new similarity-based selection multi-objective evolutionary algorithm (SSMOEA) for finding multi-objective solutions for the principal variants of the VRPB, and analyzed its performance. A general solution representation and set of evolutionary operators were designed for the basic VRPB, and modified initial solution generation and mutation procedures were created for solving several real-world VRPB variants, specifically the VRPMB, VRPSDP, VRPSB and VRPMSB. A crucial feature of SSMOEA is a novel solution similarity metric that biases the parent and survival selection processes to increase solution diversity, and that was compared with the widely-used NSGA-II [15] approach which, instead, uses a routing independent crowding distance for that purpose. The SSMOEA performance was also compared with another successful multi-objective optimizer, MOEA/D [16], which uses a rather different problem decomposition approach.

Three different objective settings of SSMOEA were tested on 62 widely-studied VRPB benchmark instances, 36 VRPMB benchmark instances, and 24 VRPSDP benchmark instances: one for minimizing the two objective functions number of routes and travel cost of VRPB, VRPMB and VRPSDP; another for minimizing the two objectives travel cost and uncollected profit of VRPSB and VRPMSB; and the third for minimizing all three objectives of VRPSB and VRPMSB. Throughout, a Pareto compliant hypervolume metric was applied to the non-dominated solutions to provide a reliable multi-objective performance indicator, and a solution diversity measure was formulated to facilitate analysis of the algorithms' operation.

Since the basic VRPB, VRPMB and VRPSDP have traditionally concentrated on minimizing only the total travel cost for a fixed number of vehicles, they have previously only been studied extensively as single objective problems, so the best solutions from SSMOEA were first compared with the single-objective results from previous approaches. It was demonstrated that, although the SSMOEA solutions are not always as good as the best-known, as one might expect given that it is actually solving a much harder problem, they are not much worse, and SSMOEA is still capable of equaling or improving upon some best-known solutions found by the earlier single-objective approaches.

The performance of SSMOEA was then compared with that of the NSGA-II crowding approach and the MOEA/D decomposition approach for the bi-objective VRPB, VRPMB and VRPSDP. A series of *t-tests* on the hypervolume and diversity values from 30 runs of each showed that, overall, there was no significant difference between the three algorithms. The conclusion was that, given the same solution representation and evolutionary operators, SSMOEA, NSGA-II, and MOEA/D perform equally well on these bi-objective problems. The bi-objective solutions from SSMOEA, NSGA-II and MOEA/D also revealed that the benchmark instances for these problems actually have very few conflicts between the two objectives (travel cost and number of routes), meaning that a more challenging bi-objective benchmark was needed to fully test the proposed SSMOEA approach.

This led to a study of the bi-objective (travel cost and uncollected profit) VRPSB and VRPMSB, which have much larger Pareto approximation sizes. Analysis of the hypervolumes for the VRPSB showed significant improvements by SSMOEA over NSGA-II in 36 out of 62 problem instances, and by SSMOEA over MOEA/D in all 62 instances. Regarding VRPMSB, there were significant improvements by SSMOEA over NSGA-II in 21 out of 36 problem instances, and by SSMOEA over MOEA/D in all 36 instances. Moreover, the solution diversities of the non-dominated sets found by SSMOEA for the VRPSB were significantly higher than those of NSGA-II for 59 of the 62 instances, and significantly higher than those of MOEA/D for all 62 instances. In the case of the VRPMSB, SSMOEA showed significant improvements in diversity over NSGA-II in 35 of the 36 instances, and over MOEA/D in all 36 instances.

For the tri-objective optimization, it was demonstrated that SSMOEA gives solutions with significantly better hypervolume than NSGA-II for 38 of the 62 VRPSB instances, and for 18 of the 36 VRPMSB instances. Moreover, solutions from SSMOEA have significantly better hypervolume than those from MOEA/D for all 62 VRPSB instances, and for 34 of the 36 VRPMSB instances. Then, it was shown that the non-dominated SSMOEA solutions are significantly more diverse than those from NSGA-II for 60 of the 62 VRPSB and 33 of the 36 VRPMSB instances, and significantly more diverse than those from MOEA/D for 61 of the 62 VRPSB and 34 of the 36 VRPMSB instances. Thus, for over half of the benchmark instances, SSMOEA performs significantly better than NSGA-II with regard to the hypervolume and diversity metrics, and for the remaining instances their performance is similar. Moreover, for the vast majority of the benchmark instances, SSMOEA performs significantly better than MOEA/D regarding hypervolume and solution diversity. It was also shown that SSMOEA and MOEA/D are much more likely than NSGA-II to find the extreme solutions where one objective is fully optimized at the expense of the other.

Analyzing the non-dominated solutions from SSMOEA when it is set to simultaneously minimize all three objectives (travel cost, number of routes, and uncollected profit), showed that it is able to achieve solutions that define an equal, or in many cases significantly larger, hypervolume than delimited by the solutions found when only two objectives are minimized. This indicates that the solutions contained in the Pareto approximations from the bi-objective optimization are enclosed in the outcome of the tri-objective optimization, which suggests a certain robustness of the proposed approach.

In conclusion, then, suitable VRPB representations and operators have been developed for the proposed SSMOEA, and these have been demonstrated to generate good Pareto approximations for a range of multi-objective VRPB variants, without seriously compromising individual solution performance compared with earlier single-objective approaches. Moreover, the proposed novel diversity-enhancing similarity-based selection procedures have been shown to provide performance enhancements over the widely used general purpose crowding approach of NSGA-II and decomposition approach of MOEA/D. Finally, the tri-objective optimization setting has proved reliable for finding a diverse Pareto approximation which spans all three objectives.

There probably remains further scope for better optimization of the evolutionary parameters (such as the population size and termination criterion) for specific VRPB variants. There is also potential for reducing the longer execution times of SSMOEA. Moreover, there are other possibilities for taking advantage of the solution similarity and diversity information, for example, in using SSMOEA on stochastic variants of the VRP. Already, though, the proposed SSMOEA shows much promise for any applications where the VRPB needs to be treated as a multi-objective problem.

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