Committee Machines

Neural Computation : Lecture 19

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What is a Committee Machine?

We have seen that it is standard practice with neural networks to train many different candidate networks, and then to select and keep the best (for example, on the basis of performance on an independent validation set) while discarding the rest.

There are two obvious disadvantages to this approach

1. All the effort involved in training the discarded networks is wasted.
2. Randomness of the noise in the data means the network with the best validation set performance will not necessarily have the best test set performance.

These drawbacks can easily be overcome by ***combining*** the networks together to form a ***committee machine***.

The importance of this approach is that it can lead to significant improvements in the performance on new data, with little extra computational effort. In fact, the committee can often do better than the best single constituent network in isolation.

Types of Committee Machine

Committee machines can be conveniently classified into two major categories:

1. Static Structures

The outputs of several constituent networks are combined by a mechanism that does not involve the input signal, hence the designation ***static***. Examples of this include

- ***Ensemble averaging***, where the constituent outputs are linearly combined.
- ***Boosting***, where weak learners are combined to give a strong learner.

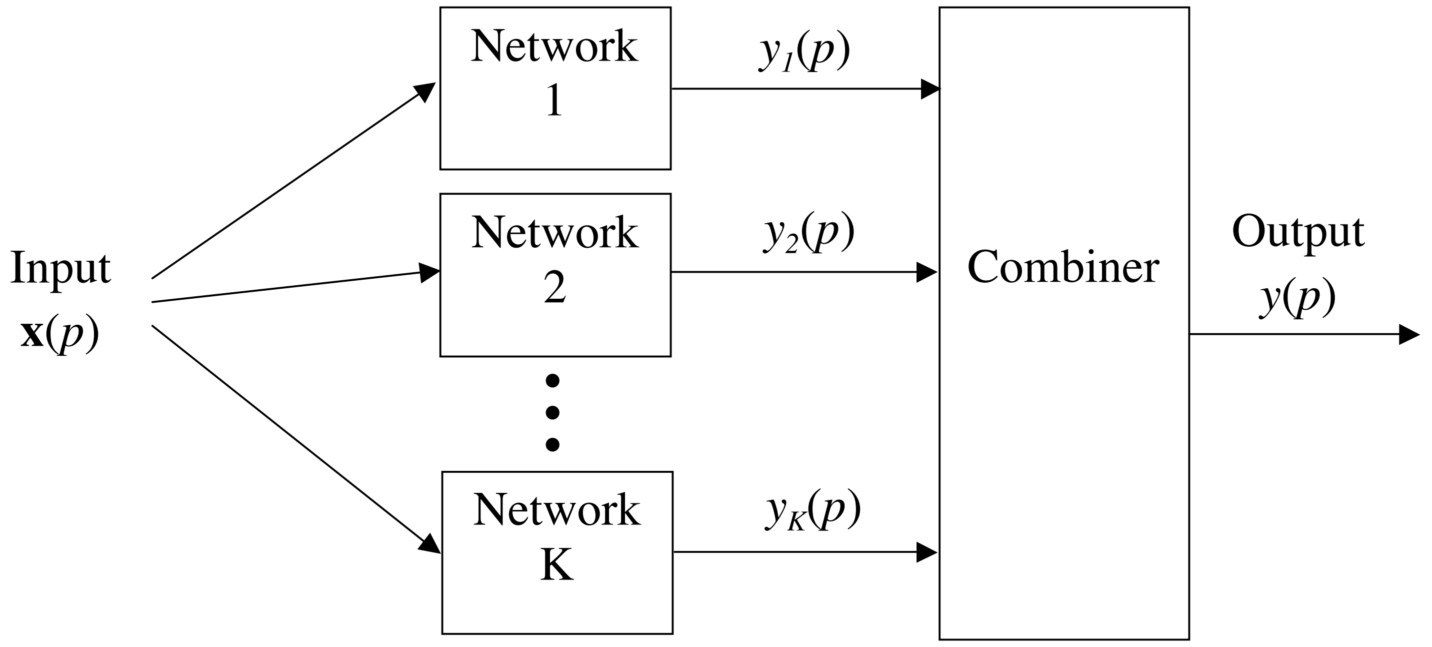
2. Dynamic Structures

The input signal is directly involved in actuating the mechanism that integrates or combines the constituent outputs, hence the designation ***dynamic***. The main example is

- ***Mixtures of experts***, where the constituent outputs are non-linearly combined by some form of gating system (which may itself be a neural network).

Ensemble Averaging

An ***Ensemble Average*** consists of a set of trained networks (e.g., MLP, RBF, or both) which share a common input $\mathbf{x}(p)$ for training pattern p, and whose individual outputs $y_i(p)$ are somehow combined to produce an overall output $y(p)$:



Usually, an output combination by simple linear averaging or voting proves sufficient.

Theory Behind Ensemble Averaging

The underlying advantage of ensemble averaging is that differently trained networks will tend to end up with different weights that correspond to different patterns of over-fitting, and hence the average performance is likely to be better than that of any individual.

There are many possible ways to make the individuals different, and many possible ways to combine their outputs. However, simply starting a set of identical networks from ***different initial weights*** and averaging their outputs tends to give good results.

A full ***bias + variance analysis*** shows that the ensemble average has the ***same bias*** as the individual networks, but the ***variance is reduced***. So, overall, this results in better generalization by the ensemble.

Studying the bias versus variance trade-off shows that the variance reducing ensemble shifts the optimal bias too, so slight over-training of the individual networks allows the ensemble to improve generalization even more.

Boosting

Boosting is quite different to ensemble averaging. The idea here is that a weak learning algorithm, which performs only slightly better than guessing, can be ***boosted*** into a strong learning algorithm by the operation of a committee machine. This is a general technique that can be used to improve the performance of ***any*** learning algorithm.

There are many variations, but the general approach of neural network based ***Boosting Machines*** is to train constituent networks on data sets with different distributions. In practice, one trains a series of networks, each one of which concentrates more strongly on the patterns learned incorrectly by the previous networks. The final output can then be obtained by combining the constituent outputs, e.g. by averaging.

The boosting algorithm can be applied repeatedly, and the error rate made arbitrarily small. There is still some debate about how well such systems can generalize. Often, with noisy data, any outliers can end up being given undue influence, and consequently the performance of this approach will suffer unless care is taken to avoid that.

Implementing Boosting

Boosting can be implemented in (at least) three fundamentally different ways:

1. Boosting by filtering

The training patterns are filtered by different versions of the weak learning algorithm. This ends up with constituent networks trained on sub-sets of the large full training set with different statistics. This tends to be the most memory efficient approach.

2. Boosting by sub-sampling

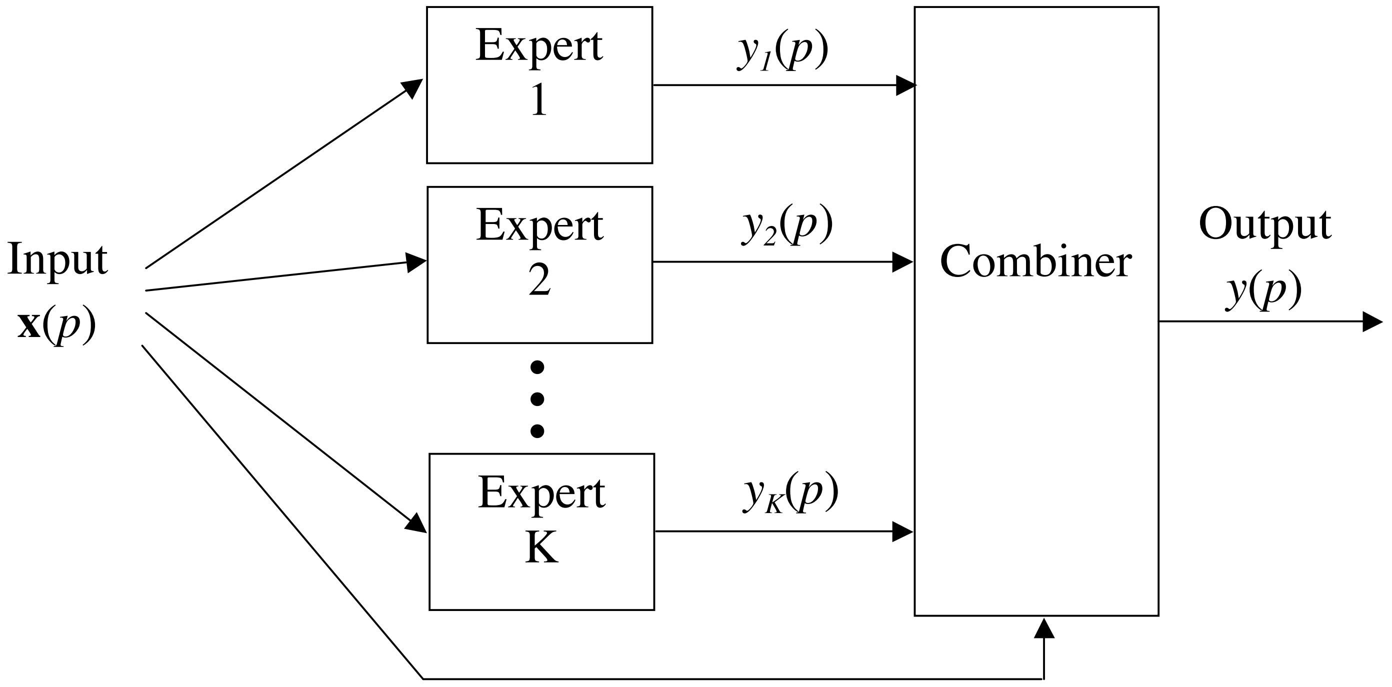
The training set is of fixed size and patterns are re-sampled according to a given probability distribution during training. In effect, the patterns learnt incorrectly get sampled more often in the next in a series of networks. The error is calculated with respect to the fixed training set. ***AdaBoost*** is a well known example of this approach.

3. Boosting by re-weighing

The training set is fixed, but the weak learning algorithm can receive "weighted" patterns. The error is calculated with respect to the weighted examples.

Mixtures of Experts

With ***Mixtures of Experts***, the principle of ***divide and conquer*** distributes the given learning task between a set of expert networks, and combines the constituent outputs to produce an overall output that is superior to that of any single network acting on its own.



Here the inputs **x**(p) influence (or gate) the combination of the constituent outputs $y_i(p)$.

Combining the Experts

Unlike with static committee machines, in a mixture of experts, each expert really can concentrate on one part of the problem and ignore the rest. The constituent outputs can be combined more intelligently so that only the right (i.e. best) experts contribute to the output for any given input pattern.

In the simple ensemble averaging approach we just take the average of the outputs

$$y(p) = \sum_{i=1}^{K} \frac{1}{K} y_i(p)$$

whereas in the mixtures of experts approach the outputs are gated according to the inputs

$$y(p) = \sum_{i=1}^{K} g_i(\mathbf{x}(p))\, y_i(p)$$

In this way the system can become truly ***modular*** with separate modules dealing with different types of inputs, or even different tasks.

Generating the Gates

In principle, the gates $g_i(\mathbf{x}(p))$ can be generated in any convenient manner. In practice, it is usual to have them produced as the output of a ***gating network***, a simple single layer network with one output for each expert. The weights a_{ji} for that network can then be learned at the same time as the weights in the expert networks. Normally, the gating network outputs are computed using the ***softmax*** activation function

$$g_i(p) = \exp\left(\sum_j a_{ij} x_j(p) \right) \Big/ \sum_{l=1}^{K} \exp\left(\sum_j a_{lj} x_j(p) \right)$$

because that automatically gives the gates useful probability-like properties:

$$0 \leq g_i(p) \leq 1 \qquad\qquad \sum_{i=1}^{K} g_i(p) = 1$$

One can then talk about weighted averages of the experts, and a gate of zero indicates the expert has no influence on the overall output, while a gate of one means total control.

Training a Mixture of Experts

Once all the gates and expert networks have been defined mathematically, it is easy to define an output error function as with any other neural network. For example

$$E(w_{jkl}, a_{ij}) = \tfrac{1}{2}\sum_p \big(t(p) - y(p)\big)^2 = \tfrac{1}{2}\sum_p \left(t(p) - \sum_j g_j(p) y_j(p) \right)^2$$

in which we have training patterns *p*, normal neural network weights w_{jkl} for each expert network *j*, weights a_{ij} for the gating network, and overall output target $t(p)$. We can then use standard gradient descent weight updates to minimise the output error function

$$\Delta w_{mnp} = -\eta \frac{\partial E(w_{jkl}, a_{ij})}{\partial w_{mnp}} \qquad , \qquad \Delta a_{mn} = -\eta \frac{\partial E(w_{jkl}, a_{ij})}{\partial a_{mn}}$$

Note that one does not need to specify targets for the gates. This mixtures of experts approach is a special case of a more general ***Mixture Model*** approach that builds overall probability distributions from a mixture of components.

Overview and Reading

1. We began by studying the basic idea of a committee machine, and the distinction between static and dynamic structures for those machines.
2. Then we looked at two types of static committee approaches: ensemble averaging and boosting. In each case, the outputs from the constituent networks are combined with no reference to the inputs.
3. We ended by looking at a dynamic approach: mixtures of experts. Here the outputs of expert networks are combined by gates which learn appropriate dependencies on the inputs. In this case, the committee can learn to take a fully modular approach to its given tasks.

Reading

1. Haykin-1999: Sections 7.1, 7.2, 7.3, 7.4, 7.5, 7.6
2. Bishop: Sections 9.6, 9.7