UNIVERSITYOF
BIRMINGHAM
School of Computer Science

First Year – Degree of BSc with Honours

Artificial Intelligence and Computer Science

Computer Science

Computer Science with Study Abroad

Computer Science with Business Management

Natural Sciences

First Year – Degree of BEng/MEng with Honours

Computer Science/Software Engineering

First Year – Joint Degree of BSc/MSci with Honours

Mathematics and Computer Science

06 18187
Foundations of Computer Science

Main Examinations 2009

Time allowed: 3 hours

[Answer ALL Questions]

[PART A. USE SEPARATE ANSWER BOOK.]

In all cases explain your methods and calculations, and justify your answers. Please make sure you read and understand each question properly before answering.

Question 1   [Arrays]   [20% in total]
Your code should be in Java or Groovy.

(a) Write a method that fills a given array of integers with the factorials $1, 1, 2, 6, 24, 120, \ldots$ until the capacity of the array is exceeded, without causing exceptions.   [10%]

(b) Write a method that rotates the elements of a given array to the right by a given number. For example, if the array is $[0, 1, 2, 3, 4, 5, 6, 7]$ and the number is 3, the resulting array should be $[5, 6, 7, 0, 1, 2, 3, 4]$. For simplicity, you may assume that the number is non-negative and smaller than the length of the array.   [10%]

Question 2   [Stacks]   [15% in total]
Implement a class for stacks using linked lists. The class should have a constructor, and implement push, pop, isEmpty methods. Your code should be in Java or Groovy, and should use private declarations wisely.

Question 3   [Computer Architecture]   [15% in total]

(a) Write the decimal number 1234 in binary.   [1%]

(b) Write the decimal number -234 in 2's complement using 16 bits. [2%]

(c) The binary numbers 10010001 and 00010001 are written in 2's complement using 8 bits. Write them in decimal.   [2%]

(d) Implement the function tabulated below using logical gates, where the inputs are C, X, Y and the outputs are Z, C'. (Notice that this is a 1-bit adder with incoming carry C and outgoing carry C'. The input digits are X and Y, and the output digit is Z.) Diagrams are not needed - logical expressions are enough.

C	X	Y	Z	C'
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

[10%]

[PART B. USE NEW ANSWER BOOK.]

Question 4   [TREES]   [17% in total]

(a) Insert the numbers 5, 4, 8, 3, 2, 7, 9 in this order in an initially empty binary search tree. Display the resulting binary search tree after each insertion. Is the resulting binary search tree perfectly balanced? Is it complete? Explain your answers in detail.   [5%]

(b) Is the following tree a binary search tree? Is it a heap tree? Is it complete? Explain your answers in detail.
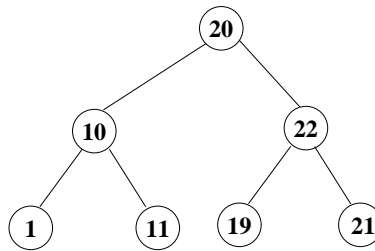
[4%]

(c) Write a procedure in pseudocode which takes as input a binary search tree (of numbers) and prints its elements in decreasing order. The basic procedures for manipulating binary trees and a `print` procedure for printing a number are given.   [4%]

(d) Let a quadtree QT be given which represents grey-scale images in the range 0 to 255. Write a procedure in pseudocode which writes all actually occurring different grey values (without repetitions) in a list.   [4%]

Question 5   [SORTING, HASH TABLES]   [16% in total]

(a) Describe insertion sort and sort the array $[9, 6, 8, 3, 7, 1]$ with it, show-
    ing each intermediate array in which a new element is inserted. For
    each intermediate array indicate which part of the array is sorted. Also
    indicate how many comparisons and how many variable assignments are
    necessary in total for each intermediate array.   [4%]

(b) Assume you have to sort an array with ten million elements. Which
    sorting algorithms would be appropriate and which ones not? Explain
    your answer in detail, mentioning time complexity.   [4%]

(c) Assume a hashtable is represented as an array of size 7. We want to
    store strings consisting of three digits. The primary hash key is the
    numerical value of the second digit modulo 7. The secondary hash key
    is the numerical value of the third digit modulo 4 increased by one.
    Insert the following strings into the initially empty hashtable: "111",
    "222", "737", "323", and "234".   [4%]

(d) The array size of a hashtable is typically chosen to be a prime number.
    Explain in detail why.   [4%]

Question 6   [GRAPHS]   [17% in total]
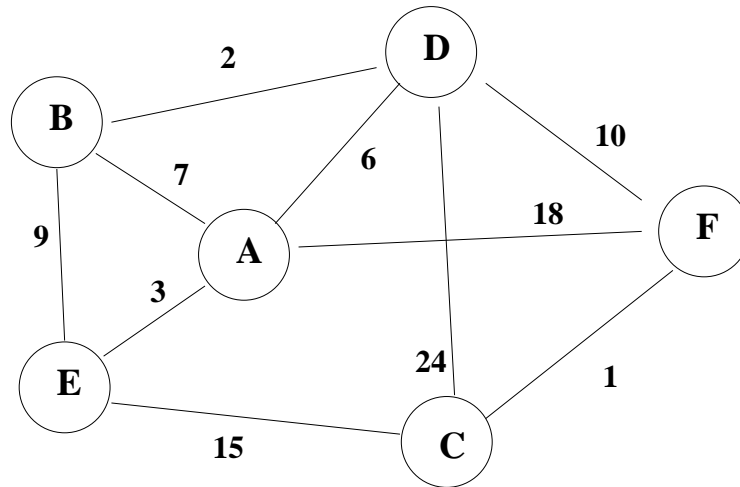
(a) Let the following graph be given:
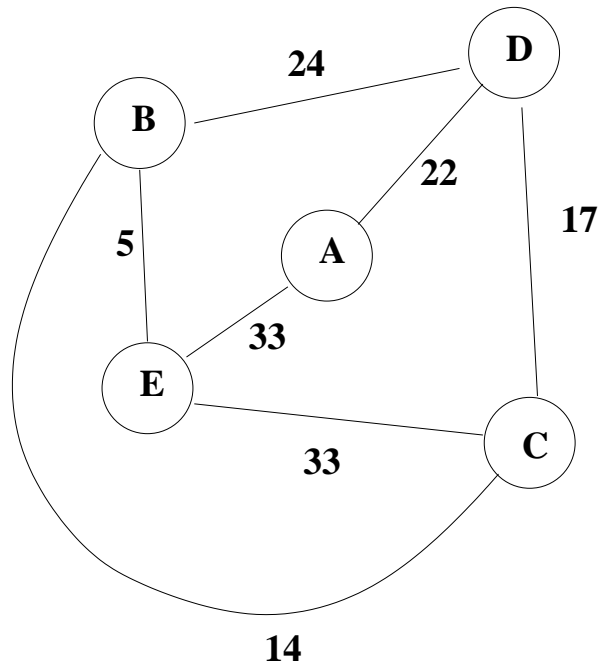
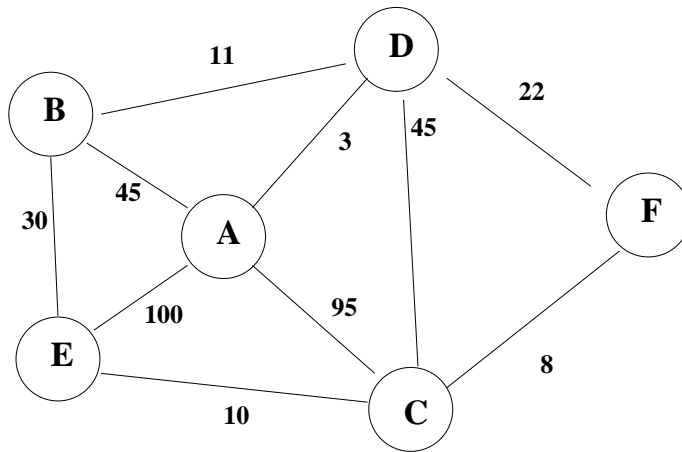What is the corresponding weight matrix?   [4%]

(b) Let the following weighted graph be given:

Apply Kruskal's algorithm to determine a minimal spanning tree. Display the graph after each step. [4%]

(c) Let the following weighted graph be given:

Use Dijkstra's algorithm to find the shortest path from **A** to **E**. What is the minimal cost? Show all relevant steps in detail. [5%]

(d) Show that Dijkstra's algorithm does not work correctly if costs can be negative. Explain your answer in detail. [4%]

End of Paper