No calculator permitted in this examination

UNIVERSITY OF BIRMINGHAM

School of Computer Science

First Year - BSc Artificial Intelligence and Computer Science
First Year - BSc Natural Sciences
First Year - BSc Computer Science
First Year - MEng Computer Science/Software Engineering

06 22754

# Foundations of Computer Science

Summer Examinations 2012

Time allowed: 3 hr

[Answer ALL Questions]

[Use the Separate Answer Book for EACH Part]

No calculator

Part A

[Use the Separate Answer Book for THIS Section]

**IN THIS SECTION YOU ARE ALLOWED TO USE ALL THE FUNCTIONS IN THE APPENDIX**

1. Write an OCaml function called `cd` which counts the number of elements which have consecutive duplicates. What is the asymptotic complexity of your function?

   For example: `cd [1; 2; 3; 3; 4; 5; 5] = 2`
   For example: `cd [1; 2; 2; 2; 3] = 2`

   [10%]

2. Using symbolic evaluation and showing all steps of computation, calculate

   ```
   map (fun n -> n+1) [1; 2]
   ```

   where `map` is defined as

   ```
   let rec map f = function
       | [ ] -> [ ]
       | x::xs -> (f x)::(map f xs)
   ```

   [10%]

3. Write an OCaml function `sf` which orders the elements of a list according to their increasing frequency. What is the asymptotic complexity of your function?

For example: `sf [1; 2; 3; 4; 2; 3; 4; 4] = [1; 2; 3; 4]`

[10%]

4. Consider this function, which "flips" a binary tree:

```
let rec flip = function
| Empty -> Empty
| Node (a, l, r) -> Node (a, flip r, flip l)
```

Using structural induction on trees show that flip is its own inverse, which means

```
flip (flip t) = t
```

for any tree **t**.

[10%]

5. (a) Explain the term “amortised complexity”.

   (b) Give the 2-list implementation of queues ("Banker's queues").

   (c) Explain why your implementation in (b) is superior to the naive implementation below:

```
Open List
type ’a queue = ’a list
let enq (x, xs) = xs @ [x]
let deq xs = (hd xs, tl xs)
```

[10%]

Part B

[Use the Separate Answer Book for THIS Section]

6. (a) Give an inductive definition of a *Binary Tree* and of a *Quad Tree.* [2%]

   (b) Explain how a Quad Tree can be used to represent a square gray-scale image. Define appropriate primitive operators for Quad Trees and use them to create an efficient pseudocode algorithm that turns a gray-scale image upside down. [6%]

   (c) What additional conditions must be satisfied for a *Binary Tree* to be a valid *Binary Search Tree*? [2%]

   (d) Draw the *Binary Search Tree* that results from inserting the items [5 9 7 3 1 8 4 2] in that order into an initially empty tree. Then draw the tree that results from removing item 5 from that tree using the standard *Binary Search Tree* delete algorithm. [4%]

7. (a) Describe in words the general idea underlying the *Quicksort* algorithm for sorting an array of items. [3%]

   (b) What are the average and worst case time complexities of the *Quicksort* algorithm? Give a simple example of how the worst case can easily arise, and how it can be avoided. [4%]

   (c) What does it mean to say that a sorting algorithm is *stable*? Discuss the stability of the *Quicksort* algorithm and the relation of stability to the computational costs of the algorithm. [3%]

   (d) Suppose you had data about seven million customers to sort according to how much they had spent, but only needed to know the fifty who had spent the most. Explain how those details would affect your choice of sorting algorithm. [3%]

8. (a) Explain what is meant by the terms *hash table*, *hash collision, linear probing and secondary hashing*. [3%]

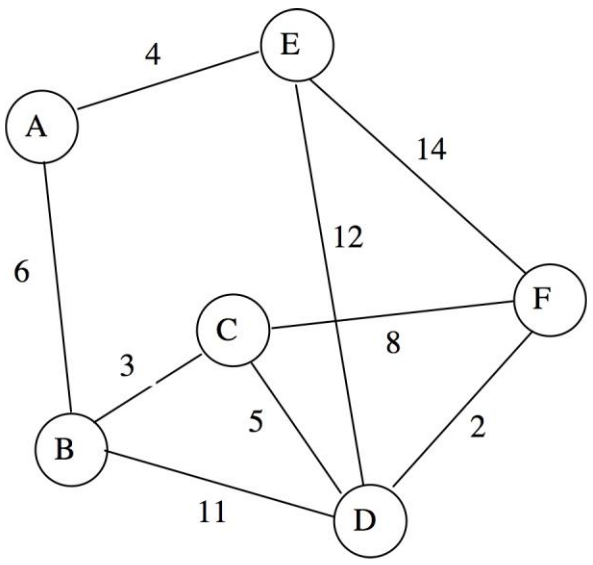
   (b) Comment on the advantages and disadvantages of using hash tables for data storage. [3%]

   (c) Suppose four digit keys are to be stored in a hash table represented as an array of size 13. The hash function is simply the first digit. Why is that not a sensible choice of hash function? Draw the initially empty hash table and insert the following keys into it using linear probing: "3487", "1346", "4143", "3571", "2583" and "0937". [3%]

   (d) Suggest an improved hash function for the above type of data and explain why it is better. [2%]

9. (a) What does it mean to say a graph is *planar*? Suggest a practical application where that property is important. Draw the graphs $K_5$ and $K_{3,3}$. Why are these graphs important in the context of planarity? [4%]

   (b) Outline the general idea of Dijkstra's algorithm for finding the shortest path between two nodes in a weighted graph. Apply the algorithm to find the shortest path from A to F in the following weighted graph, showing the computations involved at each stage. [8%]



## APPENDIX : THE LIST MODULE (SELECTED FUNCTIONS)

```
val length : 'a list -> int
```
Return the length (number of elements) of the given list.

```
val hd : 'a list -> 'a
```
Return the first element of the given list. Raise Failure "hd" if the list is empty.

```
val tl : 'a list -> 'a list
```
Return the given list without its first element. Raise Failure "tl" if the list is empty.

```
val nth : 'a list -> int -> 'a
```
Return the n-th element of the given list. The first element (head of the list) is at position 0. Raise Failure "nth" if the list is too short. Raise Invalid_argument"List.nth" if n is negative.

```
val rev : 'a list -> 'a list
```
List reversal.

```
val append : 'a list -> 'a list -> 'a list
```
Catenate two lists. Same function as the infix operator @. Not tail-recursive (length of the first argument). The @ operator is not tail-recursive either.

```
val rev_append : 'a list -> 'a list -> 'a list
```
List.rev_append l1 l2 reverses l1 and concatenates it to l2. This is equivalent to List.rev l1 @ l2, but rev_append is tail-recursive and more efficient.

```
val map : ('a -> 'b) -> 'a list -> 'b list
```
List.map f [a1; ...; an] applies function f to a1, ..., an, and builds the list [f a1; ...; f an] with the results returned by f. Not tail-recursive.

```
val rev_map : ('a -> 'b) -> 'a list -> 'b list
```
List.rev_map f l gives the same result as List.rev (List.map f l), but is tail-recursive and more efficient.

```
val fold_left : ('a -> 'b -> 'a) -> 'a -> 'b list -> 'a
```
List.fold_left f a [b1; ...; bn] is f (... (f (f a b1) b2) ...) bn.

```
val fold_right : ('a -> 'b -> 'b) -> 'a list -> 'b -> 'b
```
List.fold_right f [a1; ...; an] b is f a1 (f a2 (... (f an b) ...)). Not tail-recursive.

```
val for_all : ('a -> bool) -> 'a list -> bool
```
for_all p [a1; ...; an] checks if all elements of the list satisfy the predicate p. That is, it returns (p a1) && (p a2) && ... && (p an).

```
val exists : ('a -> bool) -> 'a list -> bool
```
exists p [a1; ...; an] checks if at least one element of the list satisfies the predicate p. That is, it returns (p a1) || (p a2) || ... || (p an).

```
val mem : 'a -> 'a list -> bool
```
mem a l is true if and only if a is equal to an element of l.

```
val find : ('a -> bool) -> 'a list -> 'a
```
find p l returns the first element of the list l that satisfies the predicate p. Raise Not_found if there is no value that satisfies p in the list l.

```
val filter : ('a -> bool) -> 'a list -> 'a list
```
filter p l returns all the elements of the list l that satisfy the predicate p. The order of the elements in the input list is preserved.

```
val find_all : ('a -> bool) -> 'a list -> 'a list
```
find_all is another name for List.filter.
```
val split : ('a * 'b) list -> 'a list * 'b list
```

Transform a list of pairs into a pair of lists: split [(a1,b1); ...; (an,bn)] is ([a1; ...; an], [b1; ...; bn]). Not tail-recursive.

```
val combine : 'a list -> 'b list -> ('a * 'b) list
```
Transform a pair of lists into a list of pairs: combine [a1; ...; an] [b1; ...; bn] is [(a1,b1); ...; (an,bn)]. Raise Invalid_argument if the two lists have different lengths. Not tail-recursive.

```
val sort : ('a -> 'a -> int) -> 'a list -> 'a list
```
Sort a list in increasing order according to a comparison function. The comparison function must return 0 if its arguments compare as equal, a positive integer if the first is greater, and a negative integer if the first is smaller (see Array.sort for a complete specification). For example, compare is a suitable comparison function. The resulting list is sorted in increasing order. List.sort is guaranteed to run in constant heap space (in addition to the size of the result list) and logarithmic stack space. The current implementation uses Merge Sort. It runs in constant heap space and logarithmic stack space.

```
val stable_sort : ('a -> 'a -> int) -> 'a list -> 'a list
```
Same as List.sort, but the sorting algorithm is guaranteed to be stable (i.e. elements that compare equal are kept in their original order) . The current implementation uses Merge Sort. It runs in constant heap space and logarithmic stack space.

```
val fast_sort : ('a -> 'a -> int) -> 'a list -> 'a list
```
Same as List.sort or List.stable_sort, whichever is faster on typical input.

```
val merge : ('a -> 'a -> int) -> 'a list -> 'a list -> 'a list
```
Merge two lists: Assuming that l1 and l2 are sorted according to the comparison function cmp, merge cmp l1 l2 will return a sorted list containting all the elements of l1 and l2. If several elements compare equal, the elements of l1 will be before the elements of l2. Not tail-recursive (sum of the lengths of the arguments).