Data Structures and Algorithms – 2018

Assignment 1 – 0% of Continuous Assessment Mark

Deadline : 5pm Monday 29th January, via Canvas

The University's *Code of Practice on Assessment and Feedback* says "Formative feedback should be provided on the first piece of work of a particular type in a programme/module". For that reason, this first assignment will be marked (with appropriate additional feedback) as usual, but the mark will not contribute towards the final mark awarded for this module.

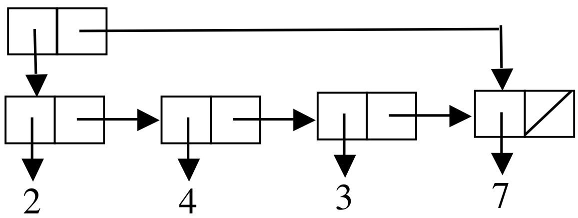
Question 1 (10 marks)

You need to insert the numbers 2, 4, 3, 7, one at a time in that order into to an initially empty queue.

Represent that process using the standard constructors `push` and `EmptyQueue`.

```
push (7, push (3, push (4, push (2, EmptyQueue))))
```

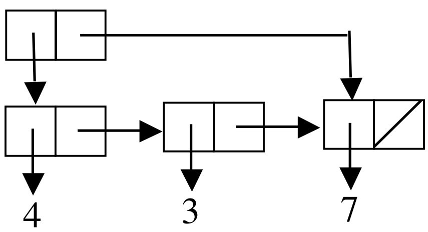
Show, in the standard two-cell notation, the resulting queue.



What is the result of the operation `top` on that queue?

2

What is the result of the operation `pop` on the original queue you created?



What is the result of the operation `pop` followed by `pop` followed by `top` on the original queue you created?

3

Question 2 (18 marks)

In the lecture notes (Section 3.2) a recursive procedure `last(L)` was defined that returns the last item in the given list `L`. By making the simplest possible modification to that procedure, create a recursive procedure `secondlast(L)` that returns the second to last item in a given list `L`.

```
secondlast(L) {
  if ( isEmpty(L) )
     error('Error: Empty list in procedure secondlast')
  elseif ( isEmpty(rest(L)) )
     error('Error: Short list in procedure secondlast')
  elseif ( isEmpty(rest(rest(L)) )
     return first(L)
  else return secondlast(rest(L))
}
```

What is the time complexity of your algorithm?

Linear in n, or O(n), where n is the length of the list.

Now carry out a more general modification of the `last(L)` procedure to give a recursive procedure `getItem(i,L)` that returns the `i`th item in the given list `L`, where `i` is an integer greater than zero. [Hint: See Lecture Notes Section 6.8.]

```
 getItem(i,L) {
   if ( isEmpty(L)) )
      error('Error: List is too short.')
   elseif  ( i == 1 )
      return first(L)
   else return getItem(i-1,rest(L))
}
```

Question 3 (10 marks)

Often one needs to check whether two given lists are the equal, i.e. contain the same items in the same order. Write a recursive procedure `equalList(L1,L2)` that returns `true` if the two given lists `L1` and `L2` are the same, and `false` if they are not. The only procedures it may call are the standard primitive list operators `first`, `rest` and `isEmpty`.

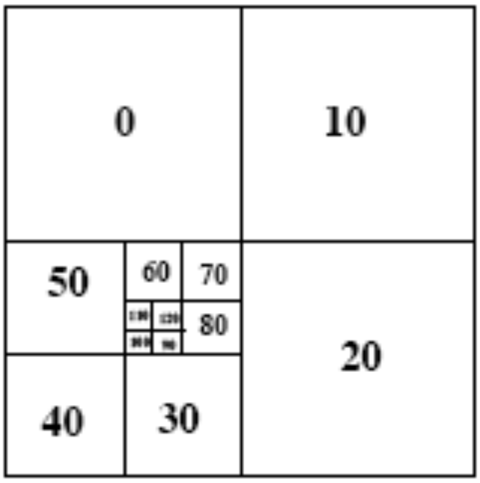
```
equalList(L1,L2) {
  if ( isEmpty(L1) and isEmpty(L2) )
      return true
  elseif ( isEmpty(L1) or isEmpty(L2) )
     return false
  elseif ( first(L1) != first(L2) )
     return false
  else return equalList(rest(L1),rest(L2))
}
```

What is the time complexity of your algorithm?

Linear in n, or O(n), where n is the length of the shortest list.

Question 4 (16 marks)

A *quadtree* was defined in the lectures in terms of primitive constructors `baseQT(value)` and `makeQT(luqt,ruqt,llqt,rlqt)`, selectors `lu(qt)`, `ll(qt)`, `ru(qt)` and `rl(qt)`, and condition `isValue(qt)`. Suppose a gray-scale picture is represented by such a quadtree with values in the range 0…255, for example:



Write a procedure `flip(qt)`, that uses the above primitive quadtree operators, to flip the picture about the vertical line through its centre.

```
flip(qt) {
   if ( isValue(qt) )
      return qt
   else return makeQT(flip(ru(qt)),flip(lu(qt)),
                      flip(rl(qt)),flip(ll(qt)) )
}
```

Write another procedure `avevalue(qt)`, that uses the above primitive quadtree operators, to return the average gray-scale value across the whole picture.

```
avevalue(qt) {
   if (isValue(qt) )
      return qt
   else return (avevalue(lu(qt)) + avevalue(ru(qt))
              + avevalue(ll(qt)) + avevalue(rl(qt)))/4
}
```

Question 5 (12 marks)

It is often important to know whether two given binary trees are the identical. Write a recursive procedure `equalBinTree(bt1,bt2)` which returns `true` if the given binary trees `bt1` and `bt2` are the same, and `false` otherwise. You can assume that you have access to the standard primitive binary tree procedures `root(bt)`, `left(bt)`, `right(bt)` and `isempty(bt)`. [Hint: Remember that you can only directly test the equality of numbers, e.g. node values.]

```
equalBinTree(t1,t2) {
   if ( isEmpty(t1) and isEmpty(t2) )
      return true
   elseif ( isEmpty(t1) or isEmpty(t2) )
      return false
   else return ( (root(t1) == root(t2) ) and
                     equalBinTree(left(t1),left(t2)) and
                        equalBinTree(right(t1),right(t2)) )
}
```

What is the time complexity of your algorithm?

Linear in n, or O(n), where n is the number of nodes in the smallest tree.

Question 6 (16 marks)

Suppose you have access to the primitive binary tree procedures `root(bt)`, `left(bt)`, `right(bt)` and `isempty(bt)`. Write a procedure `isLeaf(bt)` using them that returns `true` if the binary tree `bt` is a leaf node, and `false` if it is not.

```
isLeaf(bt) {
   if ( isempty(bt) )
      return false
   else return ( isempty(left(bt)) and isempty(right(bt)) )
}
```

Then write a recursive procedure `numLeaves(bt)` that returns the number of leaves in the given binary tree `bt`. It is only allowed to call the above primitive binary tree procedures and your `isLeaf(bt)` procedure.

```
numLeaves(bt) {
   if( isempty(bt) )
       return 0
   elseif ( isLeaf(bt) )
       return 1
   else return (numLeaves(left(bt)) + numLeaves(right(bt)))
}
```

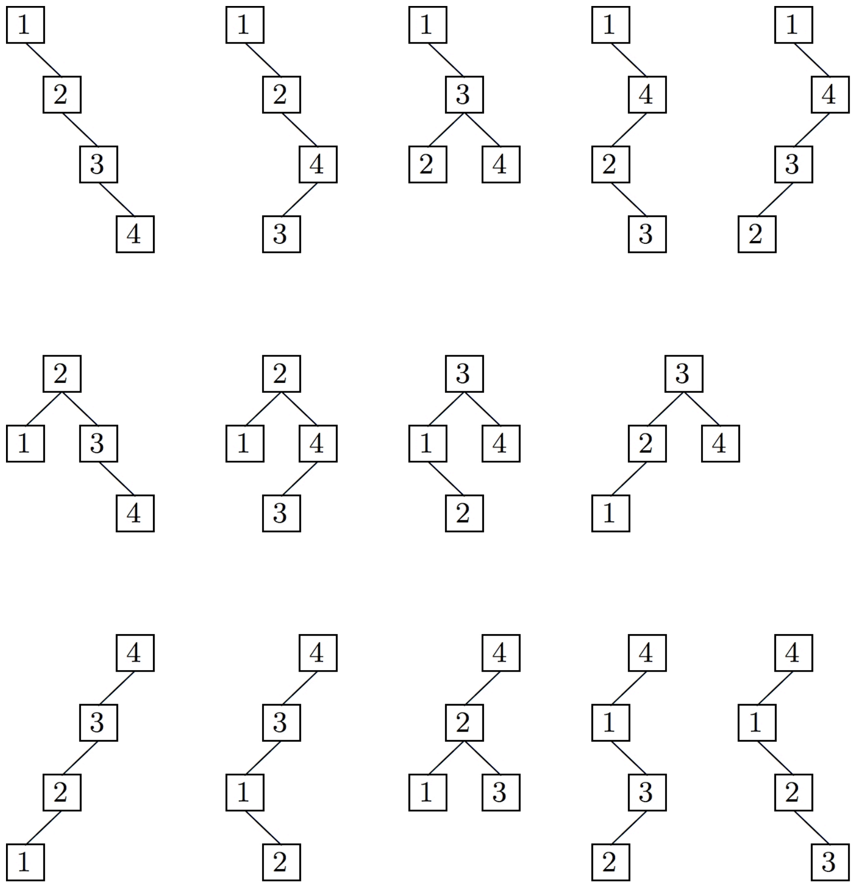
Question 7 (18 marks)

How many different orderings of the four numbers {1, 2, 3, 4} are there?

4! = 24

By considering all those possible orderings, draw all possible binary search trees of size four with nodes labeled by the four numbers {1, 2, 3, 4}. After discarding any duplicate trees, how many different binary search trees of size four are there?

Out of the 24, there are 14 different trees:



For each different tree, state its height, how many leaf nodes it has, and whether it is perfectly balanced.

Height: 3, 3, 2, 3, 3; 2, 2, 2, 2; 3, 3, 2, 3, 3.

Leafs; 1, 1, 2, 1, 1; 2, 2, 2, 2; 1, 1, 2, 1, 1.

Balanced: N, N, N, N, N; Y, Y, Y, Y; N, N, N, N, N.