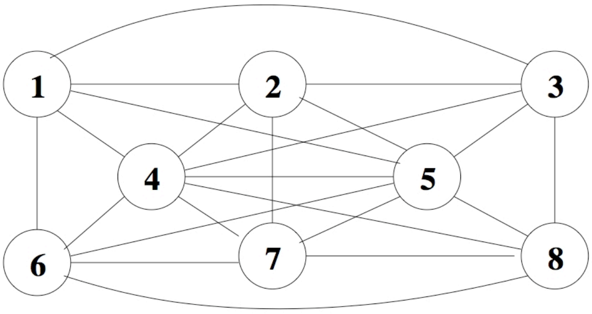
Data Structures and Algorithms – 2018

Assignment 5 – 0% of Continuous Assessment Mark

Not marked – do not submit answers to Canvas

Question 1 (16 marks)

Represent the following undirected graph as an adjacency matrix:



State in detail how depth-first traversal of the graph, starting from vertex 1, can be performed using a stack or queue. Write down the stack or queue and visited vertices at each stage.

State in detail how breadth-first traversal of the graph, starting from vertex 1, can be performed using a stack or queue. Write down the stack or queue and visited vertices at each stage.

Question 2 (12 marks)

An undirected graph is said to be *connected* if and only if for every pair of non-identical vertices there exists a path from one vertex to the other. Explain in words how you could use graph traversal to determine whether a given graph is connected.

What is the computational complexity of breadth-first and depth-first traversal in terms of the number of vertices *v* and the number of edges *e* when the graph is represented as an adjacency matrix?

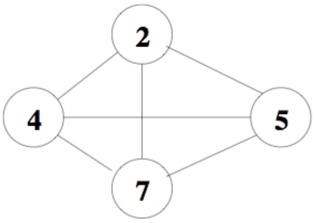
How do those computational complexities change if the graph is instead represented as an adjacency linked list?

Question 3 (14 marks)

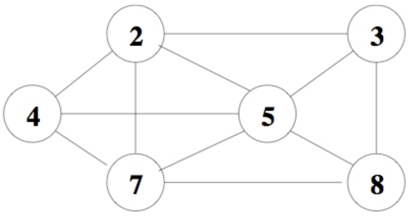
What property must a graph satisfy to be called *planar?*

Determine, without resorting to any theorems concerning K_5 and $K_{3,3}$, which of the following graphs are planar, and which are not? In each case, explain in detail how you arrived at your answer.

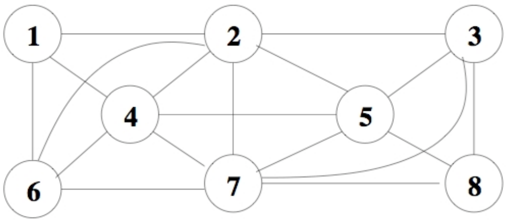
(a)



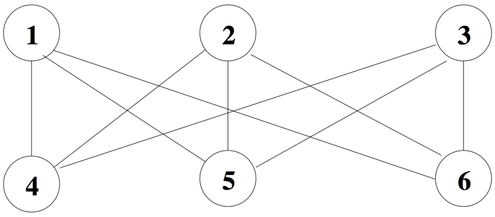
(b)



(c)

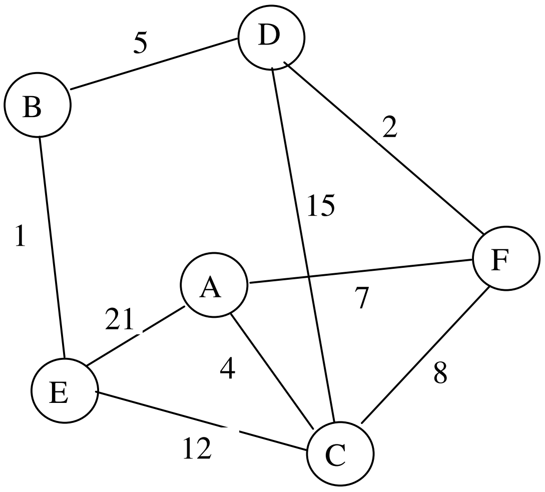


(d)



Question 4 (24 marks)

Represent the following directed graph as a weight matrix:



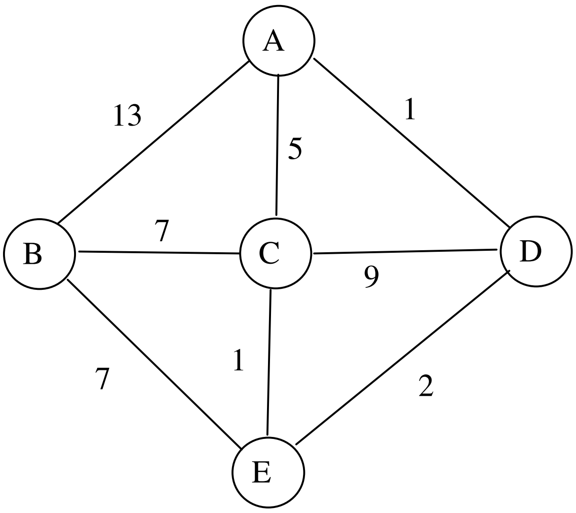
Represent the same graph as an array of adjacency lists.

Use an array-based version of Dijkstra's algorithm to determine a shortest path from vertex A to vertex E in this graph. At each stage show the estimated distances, which nodes are tight, the previous vertices, and which non-tight node has minimal estimate. At the end, explain how the shortest path is extracted from your computations, and state its length.

Explain whether Dijkstra's algorithm would work correctly on graphs that may have negative weights.

Question 5 (14 marks)

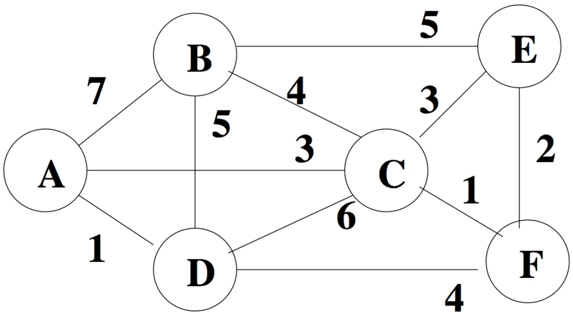
Use Floyds's algorithm to determine the lengths of the shortest paths between all vertices in the following undirected weighted graph:



Explain what you have computed at each stage.

Question 6 (20 marks)

Consider the following weighted graph:



Use Kruskal's algorithm to determine a minimal spanning tree. For each stage, show the set of edges that have been added so far.

Explain whether Kruskal's algorithm would work correctly on graphs that may have negative weights.

For the same graph, use Prim's algorithm to determine a minimal spanning tree starting from vertex A. For each stage, show the tree that have been created so far.

Explain whether Prim's algorithm would work correctly on graphs that may have negative weights.